LEARNING TARGET: I will factor polynomials completely using a variety of methods

Factoring Expressions

A **monomial** is an expression that is a number, a variable, or the product of a number and one or more variables. A **binomial**, such as $x + 4$, is the sum of two monomials. A **trinomial**, such as $x^2 + 11x + 28$, is the sum of three monomials.

A **quadratic equation** in one variable can be written in the form $ax^2 + bx + c = 0$, where $a \neq 0$. This is called the **standard form** of the equation. The solutions of a quadratic equation are called the **roots** of the equation (which are also the zeros or x-intercepts). If $ax^2 + bx + c = 0$ can be factored, then the equation can be solved using the *zero product property*.

Zero Product Property	
Words:	If the product of two expressions is zero, then one or both of the expressions equal zero
Algebra:	If A and B are expressions and AB=0, then A=0 or B=0
Example:	If $(x+5)(x+2) = 0$, then $x + 5 = 0$ or $x + 2 = 0$. That is, $x = -5$ or $x = -2$

An expression of the form $au^2 + bu + c$, where u is any expression in x, is said to be in **quadratic form**. The expression $16x^4 - 81$ is in quadratic form because it can be written as $u^2 - 81$ where $u = 4x^2$.

A factorable polynomial with integer coefficients is **factored completely** if it is written as a product of unfactorable polynomials with integer coefficients (characterized as a prime polynomial).

Factored Completely	***Not* Factored Completely**
$2(x+1)(x-4)$	$3x(x^2-4)$
$5x^2(x^2-3)$	$(4x^2+9)(4x^2-9)$

What is the difference between a factor and a zero of a function?

Zeros of functions are found when the factors of the function are set = to 0 + then the variable is solved for.

Factoring expressions often involves trial and error, however, some expressions are easy to factor because they follow special patterns.

Pattern Name	Pattern	Example
Difference of Two Squares	$a^2 - b^2 = (a+b)(a-b)$	$x^2 - 4 = (x+2)(x-2)$
Perfect Square Trinomial	$a^2 + 2ab + b^2 = (a+b)^2$ $a^2 - 2ab + b^2 = (a-b)^2$	$x^2 + 6x + 9 = (x+3)^2$ $x^2 - 4x + 4 = (x-2)^2$
Sum of Two Cubes	$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$	$8x^3 + 27 = (2x+3)(4x^2 - 6x + 9)$
Difference of Two Cubes	$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$	$64x^3 - 1 = (4x-1)(16x^2 + 4x + 1)$
Grouping	$ra + rb + sa + sb = (r+s)(a+b)$	$x^3 - 3x^2 - 16x + 48$ $= (x^2 - 16)(x-3)$ $= (x+4)(x-4)(x-3)$
GCF	A common monomial can be factored out	$4z^4 - 16z^3 + 16z^2$ $= 4z^2(z^2 - 4z + 4)$ $= 4z^2(z-2)^2$

TIPS & TRICKS

Sum & Difference of Cubes	Grouping
1. Rewrite both terms as perfect cubes 2. Write the 1st term by rewriting the problem without the cubes. 3. To write the second term use SOPS: S – Square the first term O – Opposite Sign P – Product of the terms S – Square the second term	1. Group the 1st two and the last two terms with parentheses. 2. Take out the GCF in the 1st two terms 3. Write what is left in the parentheses 4. What must be factored out of the last terms to keep the same expression in the parentheses? Factor it out. 5. The two terms on the outside of the parentheses are one factor. The expression in the parentheses is the other factor (not repeated).

Trinomials with Leading Coefficient $\neq 1$

When multiplying two binomials, you use the distributive property twice. Most of you do this without rewriting by using the FOIL process.

$$(3x-2)(4x+5)$$
$$12x^2+15x-8x-10$$
$$12x^2+7x-10$$

Notice the sum of the product of the outside terms and the product of the inside terms gives you the middle term 7x in your trinomial. The middle term is a <u>sum of the products – not a product</u>.

Example – Factor $6x^2+23x+20$

1. Find factors of the first term in the trinomial: 2x and 3x. or x and 6x These are the possible first terms in the binomial factors.
2. Find factors of the last term: Either 4 and 5 OR 2 and 10. These are the possible second terms in your binomial factors.
3. Place these factors in two binomials and see if the sum of the inside terms and the sum of the outside terms gives you the middle term, 23x.

$$(2x+4)(3x+5)$$

Product of inside terms is 12x
Product of outside terms is 10x
Is the sum 23x?

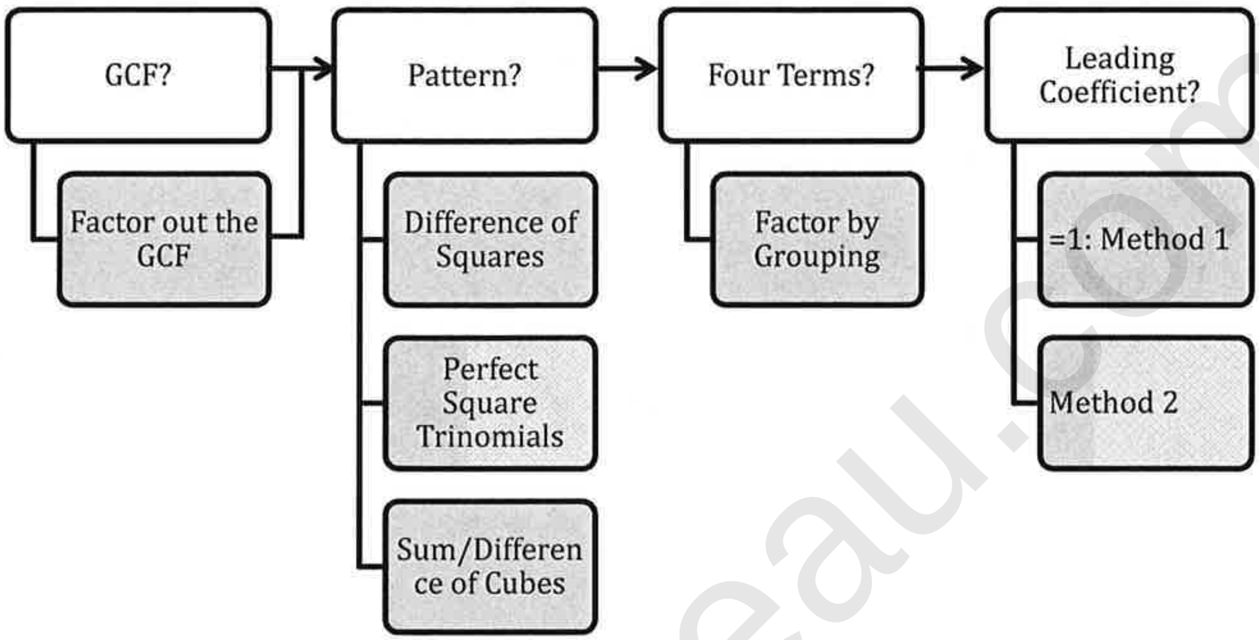
These are not our factors – before replacing 4 and 5 with 2 and 10, switch the placement of 4 and 5 and check again.

$$(2x+5)(3x+4)$$

Product of inside terms is 15x
Product of outside terms is 8x
Is the sum 23x?

We have found our factors! (2x + 5)(3x + 4) are factors of $6x^2+23x+20$

Factoring Flow Chart



Method 1 (coefficient = to 1)

- Set up parentheses & x's as first terms in each
- Sign Rule: If sign in front of c is positive then both factors have the same sign
- Ask: What 2 numbers multiply to = c and add to = b?

Method 2 (coefficient not = to 1)

- Set up parentheses
- Sign Rule: If sign in front of c is positive then both factors have the same sign
- Ask: What factors of a(c) add to = b?

EXAMPLES

Pattern Name	Examples
GCF *Pg 260*	$5x^2 - 45$ $5(x^2-9)$ $5(x+3)(x-3)$ $-5z^2 + 20z$ $-5z(z-4)$
Difference of Two Squares *Pg 253*	$x^2 - 49$ $x^2 - 7^2$ $(x+7)(x-7)$
Perfect Square Trinomial *Pg 253*	$d^2 + 12d + 36$ $d^2 + 2(d)(6) + 6^2$ $(d+6)^2$

Pattern Name	Examples
Perfect Square Trinomial (Cont) *Pg 253*	$z^2 - 26z + 169$ $z^2 - 2(z)(13) + 13^2$ $(z-13)^2$
Sum of Two Cubes *Pg 354*	$x^3 + 64$ $x^3 + (4)^3$ $(x+4)(x^2-4x+16)$ SOPS
Difference of Two Cubes *Pg 354*	$16z^5 - 250z^2$ $2z^2(8z^3 - 125)$ $2z^2[(2z)^3 - 5^3]$ SOPS $2z^2(2z-5)(4z^2+10z+25)$
Grouping *Pg 354*	$x^3 - 3x^2 - 16x + 48$ $(x^3-3x^2)(-16x+48)$ $x^2(x-3) - 16(x-3)$ $(x^2-16)(x-3)$ $\underbrace{(x+4)(x-4)}_{\text{Difference of Squares}}(x-3)$

Pattern Name	Examples
Method 1 *Pg 252*	$x^2 - 9x + 20$ $(x-5)(x-4)$ $x^2 + 3x - 12$ $(x-3)(x+6)$
Method 2 *Pg 259*	$5x^2 - 17x + 6$ 5 1 2 3 $(5x-2)(x-3)$ $-15x$ $-2x$ $-17x$ ✓ $3x^2 + 20x - 7$ 3 1 7 1 $(3x+7)(x-1)$ $\boxed{(3x-1)(x+7)}$ $-3x$ $21x$ $7x$ $-x$ $4x$ $20x$ ✓