LEARNING TARGET: I will identify properties of a normal distribution and apply those properties to determine probabilities

Calculator Capabilities

To input data into your calculator:

- Press 'STAT'
- Choose 'EDIT'
- Clear any data that already in the calculator by moving your cursor to the top of the list (where the title 'L1' is) and pressing 'CLEAR' then pressing the down arrow. Data can be cleared from any of the lists using this procedure.
- Enter all x variables in L1 by pressing the numbers desired and then pressing 'ENTER'
- Enter all y variables in L2 the same way

To graph data points entered:

- Press '2ND' and then 'Y=' (STAT PLOT)
- Choose Plot 1 by pressing 'ENTER'
- Turn Plot 1 on by placing cursor over 'ON' and pressing 'ENTER'
- Choose the type of graph by placing your cursor over your selection and pressing 'ENTER'
- Make sure 'XList' is 'L1' and 'Ylist' is 'L2.' If not, select the appropriate lists using the '2ND' and list selections in yellow text over the keys.
- Select the type of mark desired and then press 'ENTER'
- Now we must set the window by pressing 'ZOOM' and then the number '9'

To calculate descriptive statistics:

- Press '2ND' and then press 'STAT'
- Scroll over until your cursor is over the 'MATH' header
- Select the desired descriptive statistic, for example, 7: StdDev(
- Now you must select your data, so for example, enter in '2ND' and then the number '1' to select 'L1' then press 'ENTER'

To graph the normal distribution curve:

- Press 'Y=' to get to graphing screen, then press '2ND' followed by 'VARS'
- Scroll down to select '1:normalpdf(' and hit 'ENTER'
- Format is: normalpdf(x (variable), μ (mean), σ (standard deviation)) so enter in the data as such
- Now we must set the window, press 'WINDOW' and the guidelines are as follows:
  - Xmin= mean – 3SD, Xmax= mean + 3SD, Xscl = SD, Ymin = 0, Ymax = 1/(2SD), Yscl = 0
- The *area under the curve* between particular values represents the probabilities of events occurring within that specific range – we can find this area using the command 'ShadeNorm(.' Press '2ND' and then the 'VARS' key. Scroll over to 'DRAW' and select '1:ShadeNorm('
- Format is: ShadeNorm(lower bound, upper bound, mean, standard deviation) so enter data as such then press 'ENTER'
- The area of the shaded area will show on the screen – this is the probability of an event occurring within the specified range

To find the probability an event is between two values given a normal distribution:

- Press '2ND' and then 'VARS' then select '2:normalcdf('
- Format is: normalcdf(lower bound, upper bound, mean, standard deviation) so enter data as such then press 'ENTER'
  - Use -1E99 for negative infinity and 1E99 for positive infinity [1st number then 2ND Common Button 99]
- The number shown on the screen is the probability of an event occurring between these two values

Partner Statistics Review

Problem 1

You are given the data set {13, 10, 2, 2, 4, 12, 8, 6, 5, 9, 11, 14, 11, 8, 5, 8}

1. Find the mean, median, & mode.
2. What are two different data values that can be added to the set that will not affect the mean, median, and mode?
3. Using a calculator, find the standard deviation of the data set.
4. Which values are within 1 standard deviation from the mean? Are any values more than 2 standard deviations from the mean?

1. $\mu = 8$
   median = 8
   mode = 8

2. Could add one number on either side - 7 + 9

3. $3.74 = \sigma$

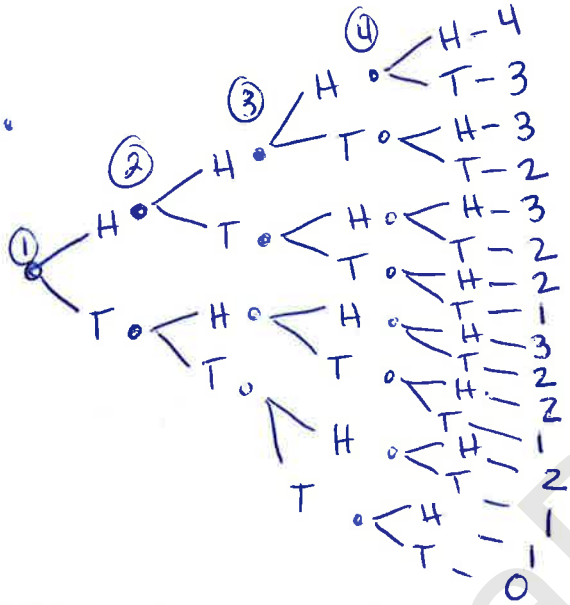
4. $4.26 \overset{1\sigma}{\frown} 11.74$
   $\{10, 8, 6, 5, 9, 11, 11, 8, 5, 8\}$
   $>2\sigma$
   < 0.52 or > 15.48
   none

Problem 2

An experiment consists of flipping four coins and recording the number of heads.

1. Draw a tree diagram showing all the possible outcomes.
2. Are all outcomes equally likely? Why or why not?
3. What is the probability of getting exactly 4 heads? Of getting 2 heads and 2 tails? Of getting 1 head and 3 tails?
4. What is the sum of all the probabilities?

1.



2. No, some outcomes have more than one way to achieve them.

3. P(4 heads) = 1/16

P(2 heads) = 6/16 = 3/8

P(1 head) = 4/16 = 1/4

4. Sum of all probabilities in tree = 1

Partner Normal Distribution Exploration

What is normal? What makes normal curves different? If you flip 10 coins 1,024 times, what is the total number of times you will get heads? Using combinations, we can obtain the expected values and theoretical probabilities show in the table below.

Number of Heads	Expected Frequency Value out of 1,024	Theoretical Probability	Percent Likelihood
0	1	$\frac{1}{1024}$	0.1%
1	10	$\frac{10}{1024}$	0.9%
2	45	$\frac{45}{1024}$	4.4%
3	120	$\frac{120}{1024}$	11.7%
4	210	$\frac{210}{1024}$	20.5%
5	252	$\frac{252}{1024}$	24.6%
6	210	$\frac{210}{1024}$	20.5%
7	120	$\frac{120}{1024}$	11.7%
8	45	$\frac{45}{1024}$	4.4%
9	10	$\frac{10}{1024}$	0.9%
10	1	$\frac{1}{1024}$	0.1%

QUESTIONS

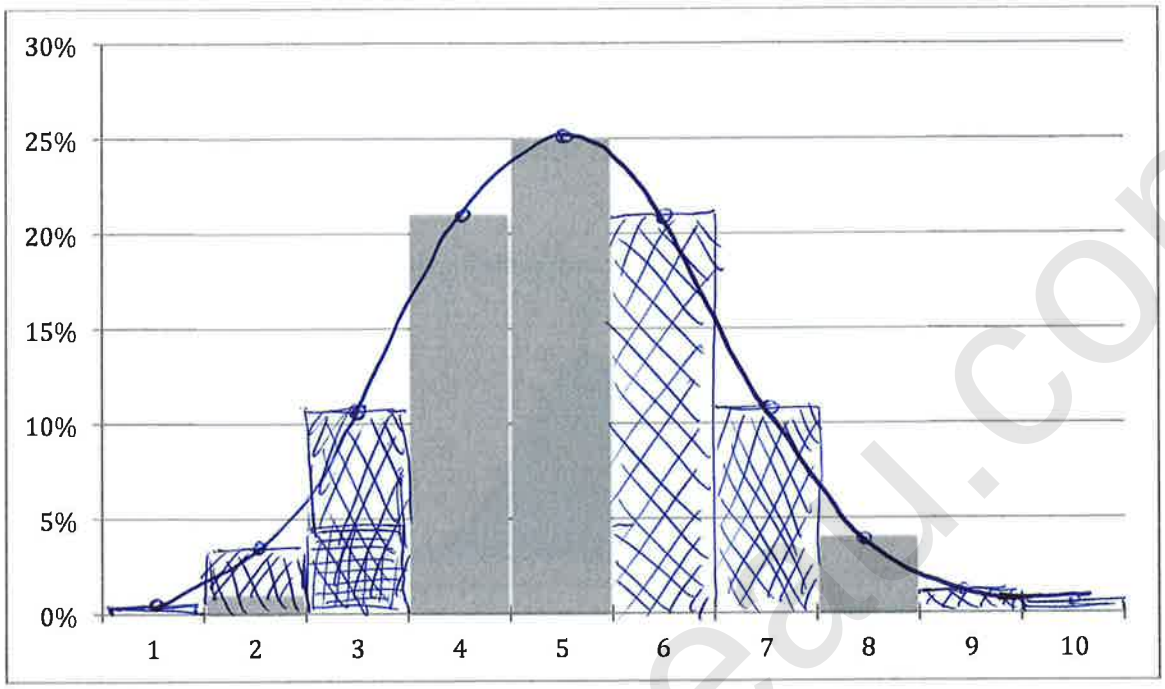
1. What is the sum of all the probabilities?

   1

2. What observations can you make about the data in the table?

   The total probability of at least one of the outcomes happening is 1 and they are symmetrically distributed

3. On the axis below, complete the histogram of the theoretical probability for each number of heads.



4. Draw a point at the midpoint of the top of each bar.
5. Connect the data points with a *smooth* curve.

6. What do you observe about the graph's shape?

Bell shaped - Symmetrical

7. What do you observe about the graph's symmetry?

Symmetrical about the middle

8. What do you observe about the graph's highest point?

Separates data into two halves.

9. What do you observe about the graph's mean/median/mode?

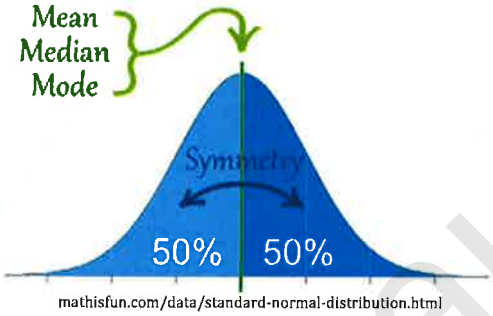
The mean, median + mode are all the same.

Normal Distribution

A **distribution** is a description of the possible values of a variable and the possible occurrences of these values.

A **normal distribution** is called a *bell curve* because its shape is comparable to a bell. If has this shape because the majority of the data is concentrated at the middle and slowly decreases symmetrically on either side.

The **standard normal distribution** is a normal distribution with $\mu = 0$ and $\sigma = 1$



All normal distributions:

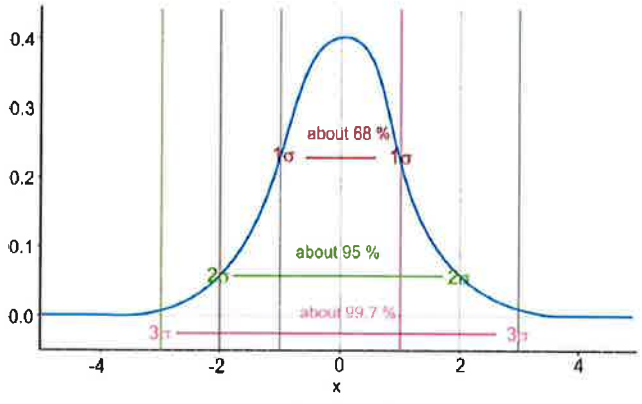
- Are perfectly symmetrical, mound-shaped distribution, also known as normal curve, or bell curve
- The curve never touches the x-axis, but it comes closer to the x-axis as it gets farther from the mean
- Center is located at the highest point over the mean μ and splits the data into two equal parts
- The mean, median, and mode are all equal
- The spread is measured with standard deviation σ, larger standard deviations mean that the data is spread farther from the center

Empirical Rule

The standard deviation is a measure of the 'typical' distance away from the mean. The space under the whole curve contains 100% of the data. Probability is calculated by find the area under the curve – the total area under the normal curve is equal to 1.

Empirical Rule:

- 68% of the data is within 1 standard deviation away from the mean
- 95% of the data is within 2 standard deviations away from the mean
- 99.7% of the data is within 3 standard deviations away from the mean



The Empirical Rule

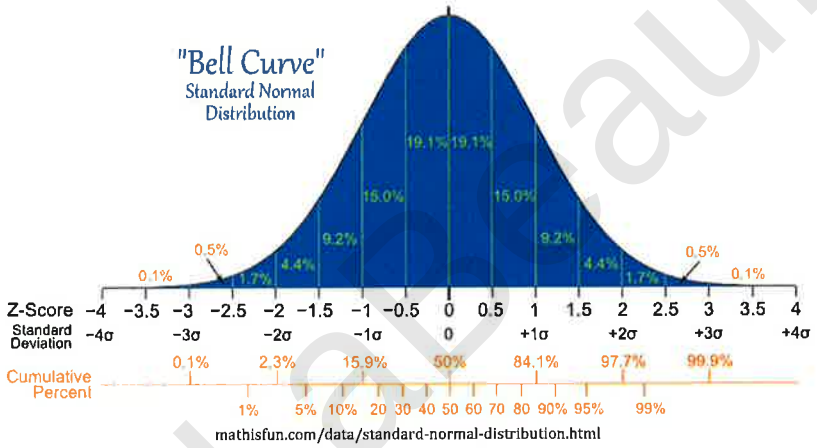
Z-Scores

A z-score tells us how many standard deviations a variable is from the mean of the population. For this reason, Z-scores are useful for comparing data from different data sets and different normal distributions. Calculating the z-scores for a data set standardizes the data so it has a normal distribution with $\mu = 0$ and $\sigma = 1$ [Standard Normal Distribution]. We can then apply the empirical rule to calculate probabilities.

To calculate a z-score, take the deviation and divide it by the standard deviation. The difference between a data value and the mean is called the deviation.

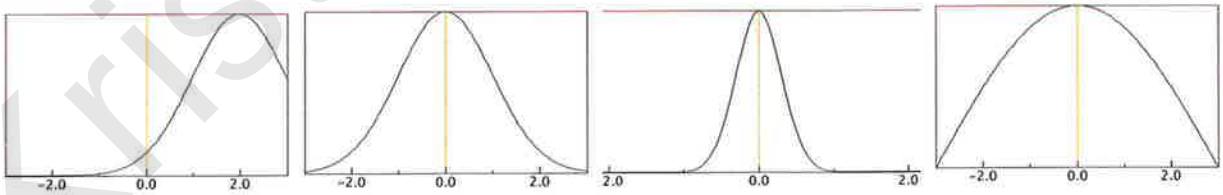
$$z = \frac{x - \mu}{\sigma}$$

σ is always positive, so if the z-score is negative, x must be below the mean.



However, the advantage of using the calculator is that it is unnecessary to standardize. We can simply enter the mean and standard deviation from the original population, avoiding the z-score calculation completely. [Function: normalcdf(on calc – detailed above]

How do the standard deviation and mean affect the graph of the normal distribution?



Mean: 2
SD: 1

Mean: 0
SD: 0.3

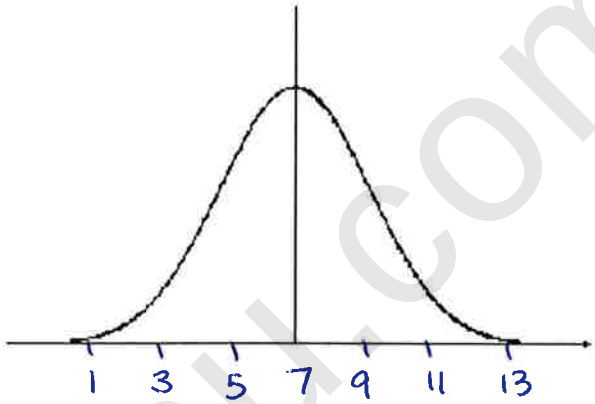
Mean: 0
SD: 1

Mean: 0
SD: 3

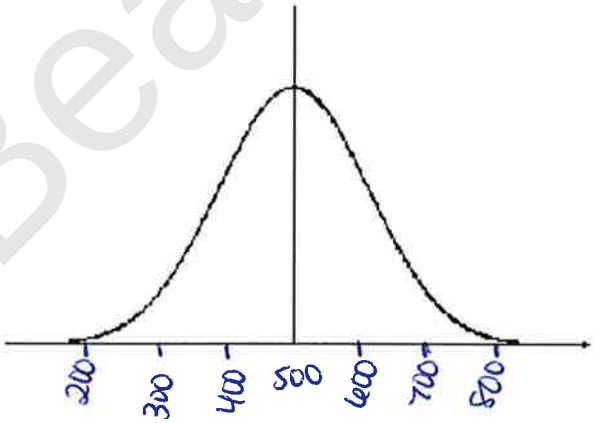
Independent Practice

Represent each of the following distributions on the normal distribution graph found to the right. For each, show three standard deviations to the left and three standard deviations to the right of the mean.

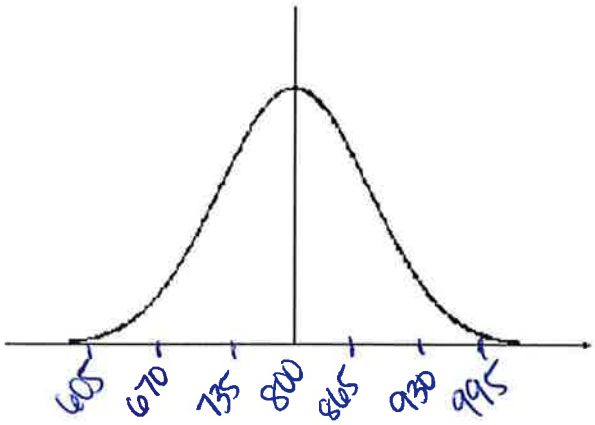
1. A normal distribution with a mean of 7 and a standard deviation of 2.



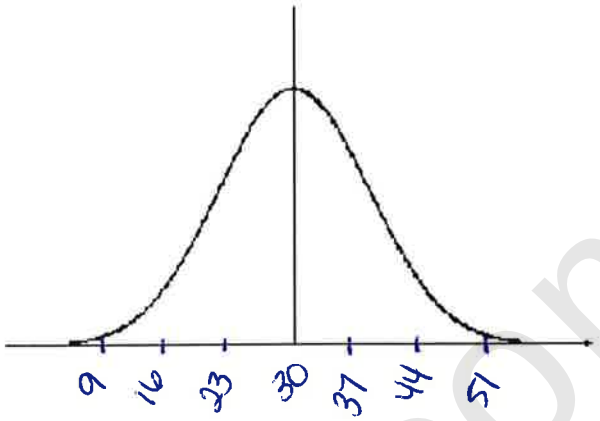
2. A normal distribution with a mean of 500 and a standard deviation of 100.



3. The weights of cattle at the fair this year were normally distributed with a mean of 800lbs and a standard deviation of 65lbs.

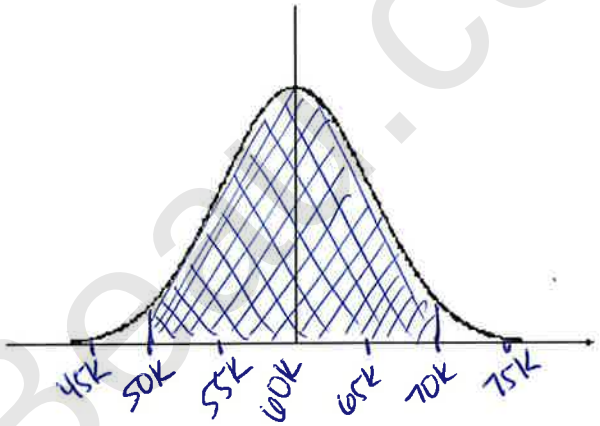


4. The amount of time a middle school student studies per night is normally distributed with a mean of 30 minutes and a standard deviation of 7 minutes.



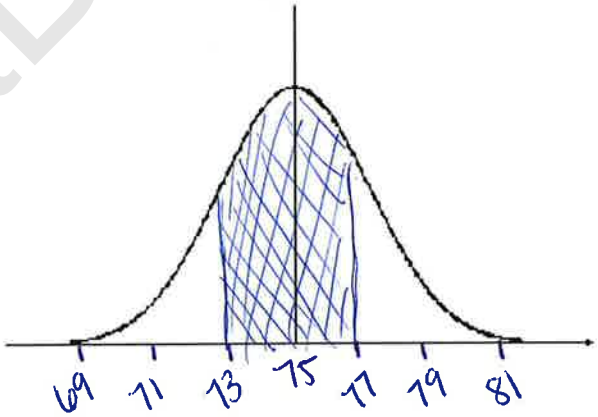
5. The length of wear on tires is normally distributed with a mean of 60,000 miles and a standard deviation of 5,000 miles. Shade the region under the curve that represents the fraction of tires that last between 50,000 miles and 70,000 miles. What percentage of tires does the shaded area represent?

95% of tires



6. The number of crackers in a box of Crackerbox Crackers is normally distributed with a mean of 75 and a standard deviation of 2. Shade the region under the curve that represents the probability that a box has between 73 and 77 crackers. What is the probability?

68%

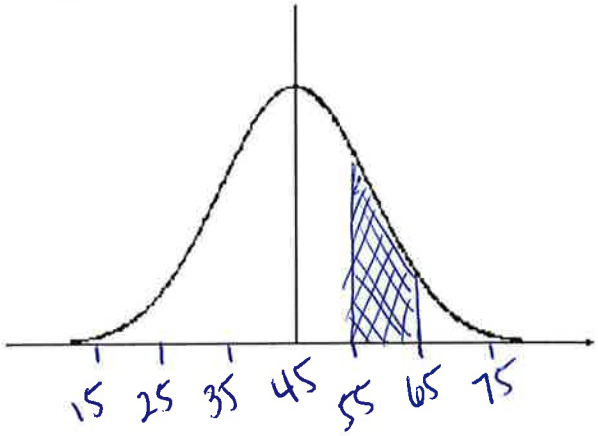


7. The length of time it takes to groom a dog at Shaggy's Pet Shop is normally distributed with a mean of 45 minutes and a standard deviation of 10 minutes. Shade the region under the curve that represents the percent of dog grooming times between 55 and 65 minutes. What is that percent?

$68 \div 2 = 34$

$95 \div 2 = 47.5$

$47.5 - 34 = 13.5\%$



Using the attached Standard Normal Probabilities Table, answer the following questions.

8. The College of Knowledge gives an admission-qualifying exam. The results are normally distributed with a mean of 500 and a standard deviation of 100. The admissions department would like to accept only students who score in the 65th percentile or better. Complete the chart below and then determine which students would qualify and what score is associated with the 65th percentile. Round to nearest hundredth in z-score

Student Score	Z-Score	Percentile	Admitted?
530	0.3	62	—
570	0.7	76	✓
650	1.5	93	✓
800	3	99.8	✓
540	.4	66	✓

* Show trick w/ calc [2ND] [ENTER] for entry, also use cdf(L, U, μ, σ) to solve.

$$z = \frac{x - \mu}{\sigma}$$

9. The MP3 player, aPod, made by Mango Corp. has an average battery of 400 hours. Battery life for the aPod is normally distributed with a standard deviation of 25 hours. The MP3 player, PeaPod, made by Pineapple Inc., has an average batter life of 390 hours. The distribution for its batter life is also normally distributed with a standard deviation of 30 hours.

MP3 Player	Average Battery Life	Standard Deviation
aPod	400	25
PeaPod	390	30

a. Find the z-scores for each battery with lives of 250, 350, 410, and 450 hours.

Z-Score Table

MP3 Player	250 hrs	350 hrs	410 hrs	450 hrs
aPod	-6	-2	$.4$	2
PeaPod	$-4.\overline{6}$	$-1.\overline{3}$	$0.\overline{6}$	2

$$Z = \frac{X - \mu}{\sigma}$$

b. Which battery lasting 410 hours performed better?

Pea pod because the Z score is higher meaning it is farther from the mean.

c. What percent of aPod batteries last between 375 and 410 hours?

50%

d. What percent of PeaPod batteries last more than 370 hours?

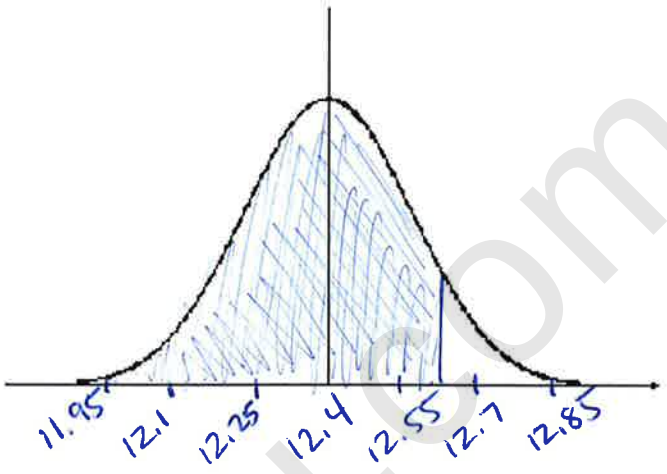
75%

10. A corn chip factory packs chips in bags with normally distributed weights with a mean of 12.4 oz. and a standard deviation of 0.15 oz.

a. On the graph to the right, label the mean and three standard deviations above and below the mean.

b. Shade the region that indicates the percentage of bags that contain less than 12.64 oz.

c. Determine the z-score corresponding to 12.64 oz.



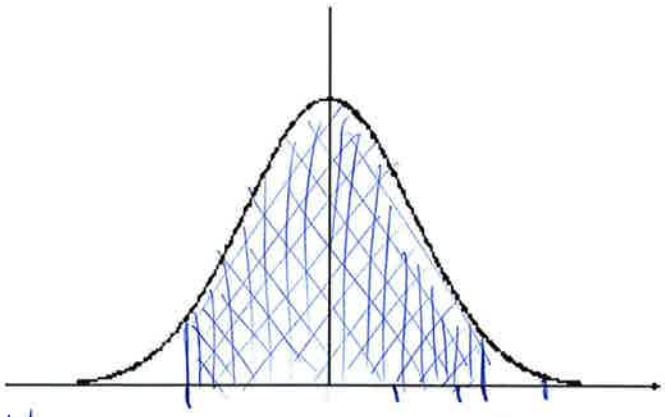
$z = \frac{12.64 - 12.4}{.15}$

$z = 1.6$

d. Find the area associated with the z-score obtained and interpret your result.

94.5% of bags have less than 12.64 oz

e. On the graph to the right, label and shade the region that represents the likelihood a bag will contain between 12.1 and 12.76 oz.



f. Calculate the z-scores corresponding to both 12.1 and 12.76 and find the probability the bag will contain between 12.1 and 12.76 oz. of chips.

$-2 = \frac{12.1 - 12.4}{.15}$ $\frac{12.76 - 12.4}{.15} = 2.4$

96.9%

g. How would you use that information to determine the probability a bag chosen at random will contain between 12.1 and 12.76 oz?

There is a 96.9% chance a bag chosen at random will contain between 12.1 + 12.76 oz.

DISCUSSION QUESTIONS

11. Why is the area under a normal curve equal to one?

The normal curve curve represents all possible outcomes and their distribution. All possible outcomes are included and the sum is equal to 1 because one of the outcomes must occur. Therefore the area under the normal curve = 1.

12. What kind of things would you look for in a data set that would indicate the set is normally distributed?

- mean, median & mode are the same #
- round shaped, symmetrical data set.

13. Describe z-score in your own words.

z-score is a metric stating how far a data point is from the mean with regards to standard deviation.

STANDARD NORMAL PROBABILITIES TABLES

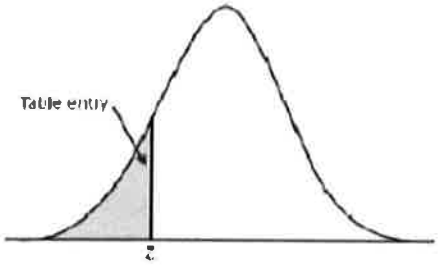


Table entry for z is the area under the standard normal curve to the left of z.

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
−3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
−3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
−3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
−3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
−3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
−2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
−2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
−2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
−2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
−2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
−2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
−2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
−2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
−2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
−2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
−1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
−1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
−1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
−1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
−1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
−1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
−1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
−1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
−1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
−1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
−0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
−0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
−0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
−0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
−0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
−0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
−0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
−0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
−0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641

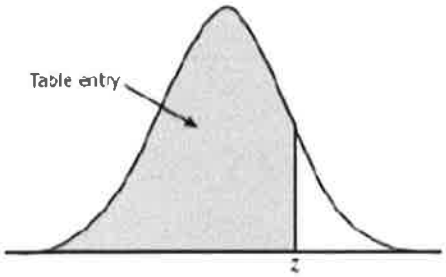


Table entry for z is the area under the standard normal curve to the left of z.

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998