

LEARNING TARGET: I will be able to solve exponential and logarithmic equations

Solving Exponential Equations

Exponential equations are equations in which variable expressions occur as exponents.

Property of Equality for Exponential Equations

If b is a positive number other than 1, then $b^x = b^y$ if and only if $x = y$

For example: If $3^x = 3^5$, then $x = 5$. If $x = 5$, then $3^x = 3^5$

There are two methods to solving exponential equations:

- Solve by equating exponents
- Take a logarithm of each side

Example 1: Solve $4^x = \left(\frac{1}{2}\right)^{x-3}$

$$(2^2)^x = (2^{-1})^{x-3}$$

$$2^{2x} = 2^{-x+3}$$

$$2x = -x + 3$$

$$\begin{array}{r} +x \quad +x \\ \hline 3x = 3 \\ \frac{3x}{3} = \frac{3}{3} \end{array}$$

$$\boxed{x=1}$$

$$4^1 = \left(\frac{1}{2}\right)^{-2}$$

$$4 = 4$$

✓

Example 2: Solve $4^x = 11$

$$\ln 4^x = \ln 11$$

$$\frac{x \ln 4}{\ln 4} = \frac{\ln 11}{\ln 4}$$

$$x = *1.73$$

*Rounded to nearest hundredth

Solving Logarithmic Equations

Logarithmic equations are equation that involve logarithms of variable expressions.

Property of Equality for Logarithmic Equations

If b, x and y are positive numbers with $b \neq 1$, then $\log_b x = \log_b y$ if and only if $x = y$

For example: If $\log_2 x = \log_2 7$, then $x = 7$. If $x = 7$, then $\log_2 x = \log_2 7$

There are two methods to solving logarithmic equations:

- Solve by equating logarithms
- Exponentiate each side of an equation

The domain of logarithmic functions do not generally include all real numbers, so you must check for extraneous solutions by checking each solution.

Example 3: Solve $\log_5(4x - 7) = \log_5(x + 5)$

$$\log_5(4x - 7) = \log_5(x + 5)$$

$$\begin{array}{r} 4x - 7 = x + 5 \\ +7 \quad +7 \end{array}$$

$$\begin{array}{r} 4x = x + 12 \\ -x \quad -x \\ \hline 3x = 12 \end{array}$$

$$\frac{3x}{3} = \frac{12}{3}$$

$$\boxed{x = 4}$$

Check

$$\log_5(9) = \log_5(9) \checkmark$$

Example 4: Solve $\log_4(5x - 1) = 3$

$$\log_4(5x - 1) = 3$$

$$4 \log_4(5x - 1) = 4^3$$

$$\begin{array}{r} 5x - 1 = 64 \\ +1 \quad +1 \end{array}$$

$$\frac{5x}{5} = \frac{65}{5}$$

$$\boxed{x = 13}$$

Check

$$\log_4(5(13) - 1) = 3$$

$$\log_4 64 = 3$$

$$4^3 = 64 \checkmark$$