LEARNING TARGET: I will be able to solve exponential and logarithmic equations

Solving Exponential Equations

Exponential equations are equations in which variable expressions occur as exponents.

> **Property of Equality for Exponential Equations**
> If b is a positive number other than 1, then $b^x = b^y$ if and only if $x = y$
> For example: If $3^x = 3^5$, then $x = 5$. If $x = 5$, then $3^x = 3^5$

There are two methods to solving exponential equations:

- Solve by equating exponents
- Take a logarithm of each side

Example 1: Solve $4^x = \left(\frac{1}{2}\right)^{x-3}$

$(2^2)^x = (2^{-1})^{x-3}$

$2^{2x} = 2^{-x+3}$

$2x = -x + 3$

$+x \quad +x$

$\frac{3x}{3} = \frac{3}{3}$

$x = 1$

$4^1 = \left(\frac{1}{2}\right)^{-2}$

$4 = 4$ ✓

Example 2: Solve $4^x = 11$

$\ln 4^x = \ln 11$

$\frac{x \ln 4}{\ln 4} = \frac{\ln 11}{\ln 4}$

$x = *1.73$

*Rounded to nearest hundreth

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Solving Logarithmic Equations

Logarithmic equations are equation that involve logarithms of variable expressions.

Property of Equality for Logarithmic Equations

If b, x and y are positive numbers with $b \neq 1$, then $\log_b x = \log_b y$ if and only if $x = y$

For example: If $\log_2 x = \log_2 7$, then $x = 7$. If $x = 7$, then $\log_2 x = \log_2 7$

There are two methods to solving logarithmic equations:

- Solve by equating logarithms
- Exponentiate each side of an equation

The domain of logarithmic functions do not generally include all real numbers, so you must check for extraneous solutions by checking each solution.

Example 3: Solve $\log_5(4x - 7) = \log_5(x + 5)$

$\log_5(4x-7) = \log_5(x+5)$

$4x - 7 = x + 5$

$+7 \quad +7$

$4x = x + 12$

$-x \quad -x$

$3x = 12$

$\frac{3x}{3} = \frac{12}{3}$

$\boxed{x = 4}$

Check

$\log_5(9) = \log_5(9)$ ✓

Example 4: Solve $\log_4(5x - 1) = 3$

$\log_4(5x-1) = 3$

$4^{\log_4(5x-1)} = 4^3$

$5x - 1 = 64$

$+1 \quad +1$

$5x = 65$

$\frac{5x}{5} = \frac{65}{5}$

$\boxed{x = 13}$

Check

$\log_4(5(13) - 1) = 3$

$\log_4 64 = 3$

$4^3 = 64$ ✓

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