LEARNING TARGET: I will simplify radical expressions and write radical expressions as expressions containing rational exponents and vice versa

Properties of Exponents & Radicals

Let a and b be real numbers and let m and n be rational numbers.

Property Name	Definition	Example
Product of Powers	$a^m \cdot a^n = a^{m+n}$	$5^{1/2} \cdot 5^{3/2} = 5^{(1/2+3/2)} = 5^2 = 25$
Power of a Power	$(a^m)^n = a^{mn}$	$3^{(5/2)^2} = 3^{(5/2 \cdot 2)} = 3^5 = 243$
Power of a Product	$(ab)^m = a^m b^m$	$(16 \cdot 9)^{1/2} = 16^{1/2} \cdot 9^{1/2} = 4 \cdot 3 = 12$
Negative Exponent	$a^{-m} = \frac{1}{a^m}, a \neq 0$	$36^{-1/2} = \frac{1}{36^{1/2}} = \frac{1}{6}$
Zero Exponent	$a^0 = 1$	$6^0 = 1$
Quotient of Powers	$\frac{a^m}{a^n} = a^{m-n}, a \neq 0$	$\frac{4^{5/2}}{4^{1/2}} = 4^{(5/2-1/2)} = 4^2 = 16$
Power of a Quotient	$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}, b \neq 0$	$\left(\frac{27}{64}\right)^{1/3} = \frac{27^{1/3}}{64^{1/3}} = \frac{3}{4}$
Product of Radicals	$\sqrt[n]{a \cdot b} = \sqrt[n]{a} \cdot \sqrt[n]{b}$	$\sqrt[3]{12} \cdot \sqrt[3]{18} = \sqrt[3]{12 \cdot 18} = \sqrt[3]{216} = 6$
Quotient of Radicals	$\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}, b \neq 0$	$\frac{\sqrt[4]{80}}{\sqrt[4]{5}} = \sqrt[4]{\frac{80}{5}} = \sqrt[4]{16} = 2$

A radical with an index n is in **simplest form** if the radicand has no perfect *nth* powers as factors and any denominator has been rationalized.

Radical expressions with the same index and radicand are **like radicals**. To add or subtract like radicals, use the distributive property.

REMEMBER: when n is odd $\sqrt[n]{x^n} = x$
when n is even $\sqrt[n]{x^n} = |x|$

Example: $\sqrt[4]{3^4} = 3$
$\sqrt[4]{(-3)^4} = 3$

Simplifying Expressions

$\sqrt[3]{135} = \sqrt[3]{27 \cdot 5}$
① Factor out perfect cube
$= \sqrt[3]{27} \cdot \sqrt[3]{5}$
$= 3\sqrt[3]{5}$

① Make denominator perfect 5th root
$\frac{\sqrt[5]{7}}{\sqrt[5]{8}} = \frac{\sqrt[5]{7}}{\sqrt[5]{8}} \cdot \frac{\sqrt[5]{4}}{\sqrt[5]{4}} = \frac{\sqrt[5]{28}}{\sqrt[5]{32}} = \frac{\sqrt[5]{28}}{2}$

$\sqrt[3]{64y^6} = \sqrt[3]{4^3(y^2)^3} = \sqrt[3]{4^3} \cdot \sqrt[3]{(y^2)^3}$
① Factor out perfect cubes
$= 4y^2$

$(27p^3q^{12})^{\frac{1}{3}} = 27^{1/3}(p^3)^{1/3}(q^{12})^{1/3}$
$3p^{(3 \cdot 1/3)}q^{(12 \cdot 1/3)} = 3pq^4$

$\frac{14xy^{\frac{1}{3}}}{2x^{\frac{3}{4}}z^{-6}} = 7x^{(1-3/4)}y^{1/3}z^{-(-6)}$
$= 7x^{1/4}y^{1/3}z^6$

$\sqrt[5]{4a^8b^{14}c^5} = \sqrt[5]{4a^5a^3b^{10}b^4c^5}$
$= \sqrt[5]{a^5b^{10}c^5} \cdot \sqrt[5]{4a^3b^4}$
$ab^2c\sqrt[5]{4a^3b^4}$

① Make denominator perfect cube
$\sqrt[3]{\frac{x}{y^8}} = \sqrt[3]{\frac{x}{y^8} \cdot \frac{y}{y}} = \sqrt[3]{\frac{xy}{y^9}} = \frac{\sqrt[3]{xy}}{y^3}$

Adding & Subtracting Radicals

$\sqrt[4]{10} + 7\sqrt[4]{10}$

$(1+7)\sqrt[4]{10} = 8\sqrt[4]{10}$

$2\left(8^{\frac{1}{5}}\right) + 10\left(8^{\frac{1}{5}}\right)$

$(2+10)(8^{1/5}) = 12(8^{1/5})$

$\sqrt[3]{54} - \sqrt[3]{2}$

$\sqrt[3]{27 \cdot 2} - \sqrt[3]{2}$

$\sqrt[3]{27} \cdot \sqrt[3]{2} - \sqrt[3]{2}$

$3\sqrt[3]{2} - \sqrt[3]{2} = (3-1)\sqrt[3]{2} = 2\sqrt[3]{2}$

$\frac{1}{5}\sqrt{w} + \frac{3}{5}\sqrt{w} = \left(\frac{1}{5} + \frac{3}{5}\right)\sqrt{w}$

$= \frac{4}{5}\sqrt{w}$

$3xy^{\frac{1}{4}} - 8xy^{\frac{1}{4}}$

$(3-8)\,xy^{1/4}$

$-5\,xy^{1/4}$

$12\sqrt[3]{2z^5} - z\sqrt[3]{54z^2}$

$12\sqrt[3]{2z^3z^2} - z\sqrt[3]{27 \cdot 2z^2}$

$12z\sqrt[3]{2z^2} - 3z\sqrt[3]{2z^2}$

$(12-3)\,z\sqrt[3]{2z^2}$

$9z\sqrt[3]{2z^2}$