

LEARNING TARGET: I will simplify radical expressions and write radical expressions as expressions containing rational exponents and vice versa

Properties of Exponents & Radicals

Let a and b be real numbers and let m and n be rational numbers.

| Property Name | Definition | Example |
|----------------------|---|--|
| Product of Powers | $a^m \cdot a^n = a^{m+n}$ | $5^{1/2} \cdot 5^{3/2} = 5^{(1/2+3/2)} = 5^2 = 25$ |
| Power of a Power | $(a^m)^n = a^{mn}$ | $3^{(5/2)^2} = 3^{(5/2 \cdot 2)} = 3^5 = 243$ |
| Power of a Product | $(ab)^m = a^m b^m$ | $(16 \cdot 9)^{1/2} = 16^{1/2} \cdot 9^{1/2} = 4 \cdot 3 = 12$ |
| Negative Exponent | $a^{-m} = \frac{1}{a^m}, a \neq 0$ | $36^{-1/2} = \frac{1}{36^{1/2}} = \frac{1}{6}$ |
| Zero Exponent | $a^0 = 1$ | $6^0 = 1$ |
| Quotient of Powers | $\frac{a^m}{a^n} = a^{m-n}, a \neq 0$ | $\frac{4^{5/2}}{4^{1/2}} = 4^{(5/2-1/2)} = 4^2 = 16$ |
| Power of a Quotient | $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}, b \neq 0$ | $\left(\frac{27}{64}\right)^{1/3} = \frac{27^{1/3}}{64^{1/3}} = \frac{3}{4}$ |
| Product of Radicals | $\sqrt[n]{a \cdot b} = \sqrt[n]{a} \cdot \sqrt[n]{b}$ | $\sqrt[3]{12 \cdot 18} = \sqrt[3]{12 \cdot 18} = \sqrt[3]{216} = 6$ |
| Quotient of Radicals | $\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}, b \neq 0$ | $\frac{\sqrt[4]{80}}{\sqrt[4]{5}} = \sqrt[4]{\frac{80}{5}} = \sqrt[4]{16} = 2$ |

A radical with an index n is in **simplest form** if the radicand has no perfect n th powers as factors and any denominator has been rationalized.

Radical expressions with the same index and radicand are **like radicals**. To add or subtract like radicals, use the distributive property.

Simplifying Expressions

REMEMBER:

when n is odd $\sqrt[n]{x^n} = x$
 when n is even $\sqrt[n]{x^n} = |x|$

Example:

$$\sqrt[4]{3^4} = 3$$

$$\sqrt[4]{(-3)^4} = 3$$

① Factor out perfect cube

$$\begin{aligned} \sqrt[3]{135} &= \sqrt[3]{27 \cdot 5} \\ &= \sqrt[3]{27} \cdot \sqrt[3]{5} \\ &= 3\sqrt[3]{5} \end{aligned}$$

① Make denominator perfect 5th root

$$\frac{\sqrt[5]{7}}{\sqrt[5]{8}} = \frac{\sqrt[5]{7}}{\sqrt[5]{4} \cdot \sqrt[5]{4}} = \frac{\sqrt[5]{7}}{\sqrt[5]{32}} = \frac{\sqrt[5]{7}}{2}$$

① Factor out perfect cubes

$$\begin{aligned} \sqrt[3]{64y^6} &= \sqrt[3]{4^3(y^2)^3} = \sqrt[3]{4^3} \cdot \sqrt[3]{(y^2)^3} \\ &= 4y^2 \end{aligned}$$

$$\begin{aligned} (27p^3q^{12})^{\frac{1}{3}} &= 27^{\frac{1}{3}} (p^3)^{\frac{1}{3}} (q^{12})^{\frac{1}{3}} \\ &= 3p \cdot q^4 = 3pq^4 \end{aligned}$$

$$\begin{aligned} \frac{14xy^{\frac{1}{3}}}{2x^{\frac{3}{4}}z^{-6}} &= 7x^{(1-\frac{3}{4})} y^{\frac{1}{3}} z^{-(-6)} \\ &= 7x^{\frac{1}{4}} y^{\frac{1}{3}} z^6 \end{aligned}$$

$$\begin{aligned} \sqrt[5]{4a^8b^{14}c^5} &= \sqrt[5]{4a^5a^3b^{10}b^4c^5} \\ &= \sqrt[5]{a^5b^{10}c^5} \cdot \sqrt[5]{4a^3b^4} \\ &= ab^2c^5 \sqrt[5]{4a^3b^4} \end{aligned}$$

① Make denominator perfect cube

$$\begin{aligned} \sqrt[3]{\frac{x}{y^8}} &= \sqrt[3]{\frac{x}{y^8} \cdot \frac{y}{y}} = \sqrt[3]{\frac{xy}{y^9}} = \frac{\sqrt[3]{xy}}{y^3} \end{aligned}$$

Adding & Subtracting Radicals

$$\sqrt[4]{10} + 7\sqrt[4]{10}$$

$$(1+7)\sqrt[4]{10} = 8\sqrt[4]{10}$$

$$2\left(8^{\frac{1}{5}}\right) + 10\left(8^{\frac{1}{5}}\right)$$

$$(2+10)\left(8^{\frac{1}{5}}\right) = 12\left(8^{\frac{1}{5}}\right)$$

$$\frac{\sqrt[3]{54} - \sqrt[3]{2}}{\sqrt[3]{27 \cdot 2} - \sqrt[3]{2}}$$

$$\frac{\sqrt[3]{27} \cdot \sqrt[3]{2} - \sqrt[3]{2}}{\sqrt[3]{27} \cdot \sqrt[3]{2} - \sqrt[3]{2}}$$

$$3\sqrt[3]{2} - \sqrt[3]{2} = (3-1)\sqrt[3]{2} = 2\sqrt[3]{2}$$

$$\frac{1}{5}\sqrt{w} + \frac{3}{5}\sqrt{w} = \left(\frac{1}{5} + \frac{3}{5}\right)\sqrt{w}$$

$$= \frac{4}{5}\sqrt{w}$$

$$3xy^{\frac{1}{4}} - 8xy^{\frac{1}{4}}$$

$$(3-8)xy^{\frac{1}{4}}$$

$$-5xy^{\frac{1}{4}}$$

$$12\sqrt[3]{2z^5} - z\sqrt[3]{54z^2}$$

$$12\sqrt[3]{2z^3z^2} - z\sqrt[3]{27 \cdot 2z^2}$$

$$12z\sqrt[3]{2z^2} - 3z\sqrt[3]{2z^2}$$

$$(12-3)z\sqrt[3]{2z^2}$$

$$9z\sqrt[3]{2z^2}$$