Simplifying Expressions with Radicals and Rational Exponents

Part I – When to switch from radicals to rational exponents

- When the roots or indices are different but the radicand is the same

Example:

$$\left(\sqrt[3]{2}\bullet\sqrt[6]{2}\right)^4 \rightarrow \left(2^{\frac{1}{3}}\bullet 2^{\frac{1}{6}}\right)^4 \rightarrow \left(2^{\frac{1}{2}}\right)^4 \rightarrow 2^{\frac{4}{2}} = 2^2 = 4$$

Simplify the following:

1. $\left(\sqrt[3]{4}\bullet\sqrt[6]{4}\right)^3$

$(4^{1/3}\cdot 4^{1/6})^3$

$(4^{1/2})^3$

$\boxed{4^{3/2}}$

2. $\left(\sqrt{5}\bullet\sqrt[4]{5}\right)^8$

$(5^{1/2}\cdot 5^{1/4})^8$

$(5^{3/4})^{8^2}$

$\boxed{5^6}$

3. $\left(\sqrt[3]{3}\bullet\sqrt[6]{9}\right)^6$

$(3^{1/3}\cdot 9^{1/6})^6$

$(3^{1/3}\cdot(3^2)^{1/6})^6$

$(3^{1/3}\cdot 3^{2/6})^6$

$(3^{4/6})^6$

$\boxed{3^4}$

4. $\dfrac{\sqrt[3]{3}}{\sqrt[4]{3}}$

$\dfrac{3^{1/3}}{3^{1/4}} = 3^{1/3-1/4}$

$3^{\frac{4}{12}-3/12} = \boxed{3^{1/12}}$

5. $\dfrac{\sqrt[4]{2}}{\sqrt[3]{4}}$

$\dfrac{2^{1/4}}{4^{1/3}} = \dfrac{2^{1/4}}{(2^2)^{1/3}} = \dfrac{2^{1/4}}{2^{2/3}}$

$2^{1/4-2/3} = 2^{3/12-8/12}$

$\boxed{2^{-5/12}}$

6. $\dfrac{\sqrt[3]{9}}{\sqrt[5]{3}} = \dfrac{9^{1/3}}{3^{1/5}}$

$\dfrac{(3^2)^{1/3}}{3^{1/5}} = \dfrac{3^{2/3}}{3^{1/5}}$

$= 3^{2/3-1/5}$

$= 3^{10/15-3/15}$

$= \boxed{3^{7/15}}$

Part II – Simplifying expressions with rational exponents

- When the base is same – subtract the exponents

Example

$$\frac{3^{\frac{1}{3}}}{3^{-\frac{1}{4}}} \rightarrow 3^{\frac{7}{12}}$$

- If the bases are different:
  - See you can manipulate one base to be the same and then subtract
  - Rationalize the denominator and simplify

Example

Method 1:

- Remember – a fraction exponent represents a radical and radicals cannot remain in the denominator so the fraction exponent needs to be removed.
- Multiply the numerator and denominator by $3^{\frac{3}{4}}$ because $3^{\frac{1}{4}} \bullet 3^{\frac{3}{4}} = 3^{\frac{4}{4}} = 3$ and the fraction exponent is removed.

$$\frac{16^{\frac{1}{4}}}{3^{\frac{1}{4}}} \rightarrow \frac{2 \bullet 3^{\frac{3}{4}}}{3}$$

Method 2:

- Re-write the numerator and denominator in radical form.
- The fourth root of 16 is 2.
- The radical must be removed from the denominator so we must multiply by a number that removes the radical.

$$\frac{16^{\frac{1}{4}}}{3^{\frac{1}{4}}} \rightarrow \frac{\sqrt[4]{16}}{\sqrt[4]{3}} \rightarrow \frac{2}{\sqrt[4]{3}} \rightarrow \frac{2}{\sqrt[4]{3}} \bullet \frac{\sqrt[4]{3^3}}{\sqrt[4]{3^3}} \rightarrow \frac{2\sqrt[4]{3^3}}{3} = \frac{2\sqrt[4]{27}}{3}$$

Simplify the following:

7. $\dfrac{4^{\frac{4}{7}}}{4^{-\frac{1}{2}}}$

$= 4^{4/7 - -1/2}$

$= 4^{4/7 + 1/2}$

$= 4^{8/14 + 7/14}$

$= \boxed{4^{15/14}}$

8. $\dfrac{3^{\frac{1}{3}}}{8^{\frac{1}{3}}} = \dfrac{3^{1/3}}{(2^3)^{1/3}}$

$= \dfrac{3^{1/3}}{2}$

$= \boxed{\dfrac{\sqrt[3]{3}}{2}}$

9. $\dfrac{32^{\frac{1}{5}}}{8^{\frac{1}{5}}} = \dfrac{(2^5)^{1/5}}{(2^3)^{1/5}}$

$= \dfrac{2^{5/5}}{2^{3/5}} \cdot \dfrac{2^{2/5}}{2^{2/5}}$

$= \dfrac{2^{7/5}}{2^{5/5}} = 2^{7/5 - 5/5}$

$\boxed{= 2^{2/5}}$

10. $\left(x^{\frac{1}{3}}y^{\frac{2}{5}}\right)^4 + 5\left(x^{\frac{5}{15}}y^{\frac{6}{15}}\right)^4$

$x^{4/3} y^{8/5} + 5\left(x^{\cancel{20/15}\,4/3} y^{\cancel{24/15}\,8/5}\right)$

$x^{4/3} y^{8/5} + 5x^{4/3} y^{8/5}$

$x^{4/3} y^{8/5} (1+5)$

$\boxed{6x^{4/3} y^{8/5}}$

Part III – Simplifying radicals

- If the expression is a fraction, simplify the fraction under the radical as much as possible.
- All factors must be represented using prime numbers.
- The exponents of the prime numbers or variables must be less than or equal to the root or index of the radical.
- If the root and the power of a number/ variable are the same, that variable or number must be brought out of the radical.

Example

- Simplify the fraction under the radical
- Represent all numbers using prime factors
- Rewrite numbers/variables if exponents are greater than the root which in this case is 2 (the square root)
- Remove all factors with an exponent of 2
- Remove the radical from the denominator – under the radical, z must have an exponent of 2 so multiply the numerator and denominator by $\sqrt{z}$, which will give us our final result

$$\sqrt{\frac{20x^3y^2}{9xz^3}} \rightarrow \sqrt{\frac{20x^2y^2}{9z^3}} \rightarrow \sqrt{\frac{2^2 \bullet 5x^2y^2}{3^2z^3}} \rightarrow \sqrt{\frac{2^2 \bullet 5x^2y^2}{3^2z^2z^1}} \rightarrow \frac{2xy\sqrt{5}}{3z\sqrt{z}} \rightarrow \frac{2xy\sqrt{5}}{3z\sqrt{z}} \bullet \frac{\sqrt{z}}{\sqrt{z}} \rightarrow \frac{2xy\sqrt{5z}}{3z^2}$$

Simplify the following:

11. $\frac{\sqrt[4]{32}}{\sqrt[4]{2}} = \sqrt[4]{16}$

$= \boxed{2}$

12. $\frac{\sqrt[5]{5}}{\sqrt[5]{27}} = \frac{5^{1/5}}{27^{1/5}} \cdot \frac{27^{4/5}}{27^{4/5}}$

$= \frac{5^{1/5} \cdot 27^{4/5}}{27}$

$= \frac{\sqrt[5]{5}\left(\sqrt[5]{27}\right)^4}{27}$

13. $\sqrt[3]{16x^4}$

$= \sqrt[3]{(2)^4 x^4}$

$= \sqrt[3]{2^3\cdot 2\cdot x^3\cdot x}$

$= 2x\sqrt[3]{2x}$

14. $\dfrac{\sqrt[3]{64x^3y}}{4x^{-3}y} = \dfrac{\sqrt[3]{(4)^3x^3 y}}{4x^{-3}y}$

$= \dfrac{\cancel{4}x\sqrt[3]{y}}{\cancel{4}x^{-3}y} = \dfrac{x^3\cdot x^3\sqrt[3]{y}}{y}$

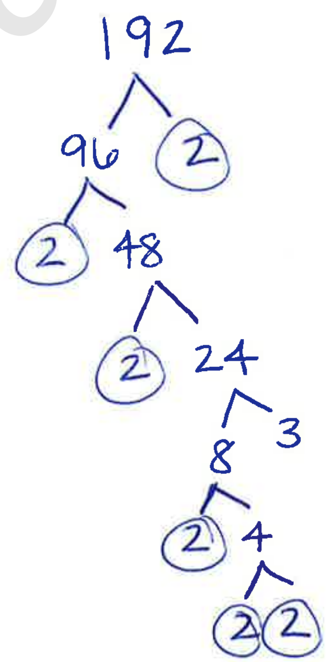
$\boxed{= \dfrac{x^6\sqrt[3]{y}}{y}}$

15. $\dfrac{\sqrt[4]{192x^3y^7}}{\sqrt[4]{6xy^{-2}}} = \dfrac{\sqrt[4]{2^6\cdot 3\cdot x^3\cdot y^4\cdot y^3}}{\sqrt[4]{2\cdot 3\cdot x\cdot y^{-2}}}$

$= \sqrt[4]{\dfrac{\cancel{2^6}\cdot\cancel{3}\cdot\cancel{x^3}\cdot y^4\cdot y^3\cdot y^2}{\cancel{2}\cdot\cancel{3}\cdot\cancel{x}}}$

$= \sqrt[4]{2\cdot 2^4\cdot x^2\cdot y^4\cdot y^4\cdot y}$

$\boxed{= 2y^2\sqrt[4]{2x^2y}}$



Part IV – Like Radicals

- Like radicals have the same radicand and the same index.
- If you have like radicals, then they may be added and/or subtracted just like any other like term. Before you do anything with these expressions, simplify each term individually to its simplest form.
- Then, you treat the radicals like you would any variable … when you are adding or subtracting, you are "counting" how many like radicals you have.

Example
- Simplify the second term by representing 512 as prime factors.
- Now you have "like radicals". How many do you have?

$$12\sqrt[4]{2}-7\sqrt[4]{512} \rightarrow 7\sqrt[4]{512}=7\sqrt[4]{2^9}=7\bullet 2\bullet 2\sqrt[4]{2}=28\sqrt[4]{2} \rightarrow 12\sqrt[4]{2}-28\sqrt[4]{2} \rightarrow -16\sqrt[4]{2}$$

Simplify the following:

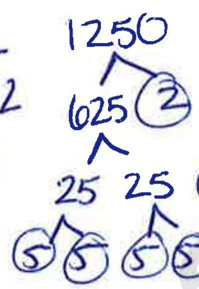
16. $2\sqrt[4]{1250}-8\sqrt[4]{32}$

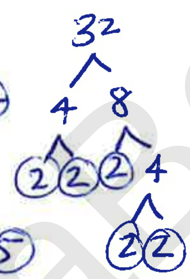
$2\sqrt[4]{5^4\cdot 2}-8\sqrt[4]{2^4\cdot 2}$

$2\cdot 5\sqrt[4]{2}-2\cdot 8\sqrt[4]{2}$

$10\sqrt[4]{2}-16\sqrt[4]{2}$

$\boxed{-6\sqrt[4]{2}}$





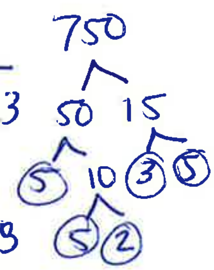
17. $5\sqrt[3]{48}-\sqrt[3]{750}$

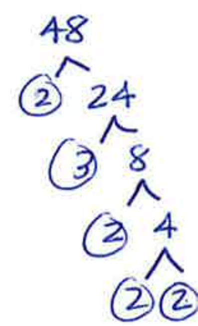
$5\sqrt[3]{2^4\cdot 3}-\sqrt[3]{5^3\cdot 2\cdot 3}$

$5\cdot 2\sqrt[3]{2\cdot 3}-5\sqrt[3]{2\cdot 3}$

$10\sqrt[3]{6}-5\sqrt[3]{6}$

$\boxed{5\sqrt[3]{6}}$





18. $\frac{3}{5}\sqrt[3]{5}-\frac{1}{5}\sqrt[3]{625}$

$\frac{3}{5}\sqrt[3]{5}-\frac{1}{5}\sqrt[3]{5^4}$

$\frac{3}{5}\sqrt[3]{5}-\sqrt[3]{5}$

$\boxed{-\frac{2}{5}\sqrt[3]{5}}$