

Simplifying Expressions with Radicals and Rational Exponents

Part I - When to switch from radicals to rational exponents

- When the roots or indices are different but the radicand is the same

Example:

$$(\sqrt[3]{2} \cdot \sqrt[6]{2})^4 \rightarrow \left(2^{\frac{1}{3}} \cdot 2^{\frac{1}{6}}\right)^4 \rightarrow \left(2^{\frac{1}{2}}\right)^4 \rightarrow 2^{\frac{4}{2}} = 2^2 = 4$$

Simplify the following:

1. $(\sqrt[3]{4} \cdot \sqrt[6]{4})^3$

$$(4^{1/3} \cdot 4^{1/6})^3$$

$$(4^{1/2})^3$$

$$\boxed{4^{3/2}}$$

2. $(\sqrt{5} \cdot \sqrt[4]{5})^8$

$$(5^{1/2} \cdot 5^{1/4})^8$$

$$(5^{3/4})^8$$

$$\boxed{5^6}$$

3. $(\sqrt[3]{3} \cdot \sqrt[6]{9})^6$

$$(3^{1/3} \cdot 9^{1/6})^6$$

$$(3^{1/3} \cdot (3^2)^{1/6})^6$$

$$(3^{1/3} \cdot 3^{2/6})^6$$

$$(3^{4/6})^6$$

$$\boxed{3^4}$$

4. $\frac{\sqrt[3]{3}}{\sqrt[4]{3}}$

$$\frac{3^{1/3}}{3^{1/4}} = 3^{1/3 - 1/4}$$

$$3^{4/12 - 3/12}$$

$$= \boxed{3^{1/12}}$$

5. $\frac{\sqrt[4]{2}}{\sqrt[3]{4}}$

$$\frac{2^{1/4}}{4^{1/3}} = \frac{2^{1/4}}{(2^2)^{1/3}} = \frac{2^{1/4}}{2^{2/3}}$$

$$2^{1/4 - 2/3} = 2^{3/12 - 8/12}$$

$$\boxed{2^{-5/12}}$$

6. $\frac{\sqrt[3]{9}}{\sqrt[5]{3}} = \frac{9^{1/3}}{3^{1/5}}$

$$\frac{(3^2)^{1/3}}{3^{1/5}} = \frac{3^{2/3}}{3^{1/5}}$$

$$= 3^{2/3 - 1/5}$$

$$= 3^{10/15 - 3/15}$$

$$= \boxed{3^{7/15}}$$

Part II – Simplifying expressions with rational exponents

- When the base is same – subtract the exponents

Example

$$\frac{3^{\frac{1}{3}}}{3^{-\frac{1}{4}}} \rightarrow 3^{\frac{7}{12}}$$

- If the bases are different:
 - See you can manipulate one base to be the same and then subtract
 - Rationalize the denominator and simplify

Example

Method 1:

- Remember – a fraction exponent represents a radical and radicals cannot remain in the denominator so the fraction exponent needs to be removed.
- Multiply the numerator and denominator by $3^{\frac{3}{4}}$ because $3^{\frac{1}{4}} \cdot 3^{\frac{3}{4}} = 3^{\frac{4}{4}} = 3$ and the fraction exponent is removed.

$$\frac{16^{\frac{1}{4}}}{3^{\frac{1}{4}}} \rightarrow \frac{2 \cdot 3^{\frac{3}{4}}}{3}$$

Method 2:

- Re-write the numerator and denominator in radical form.
- The fourth root of 16 is 2.
- The radical must be removed from the denominator so we must multiply by a number that removes the radical.

$$\frac{16^{\frac{1}{4}}}{3^{\frac{1}{4}}} \rightarrow \frac{\sqrt[4]{16}}{\sqrt[4]{3}} \rightarrow \frac{2}{\sqrt[4]{3}} \rightarrow \frac{2}{\sqrt[4]{3}} \cdot \frac{\sqrt[4]{3^3}}{\sqrt[4]{3^3}} \rightarrow \frac{2\sqrt[4]{3^3}}{3} = \frac{2\sqrt[4]{27}}{3}$$

Simplify the following:

$$\begin{aligned} 7. \quad & \frac{4^{\frac{4}{7}}}{4^{\frac{1}{2}}} \\ &= 4^{4/7 - 1/2} \\ &= 4^{4/7 + 1/2} \\ &= 4^{8/14 + 7/14} \\ &= \boxed{4^{15/14}} \end{aligned}$$

$$\begin{aligned} 8. \quad & \frac{3^{\frac{1}{3}}}{8^{\frac{1}{3}}} = \frac{3^{1/3}}{(2^3)^{1/3}} \\ &= \frac{3^{1/3}}{2} \\ &= \boxed{\frac{\sqrt[3]{3}}{2}} \end{aligned}$$

$$\begin{aligned} 9. \quad & \frac{32^{\frac{1}{5}}}{8^{\frac{1}{5}}} = \frac{(2^5)^{1/5}}{(2^3)^{1/5}} \\ &= \frac{2^{5/5}}{2^{3/5}} \cdot \frac{2^{2/5}}{2^{2/5}} \\ &= \frac{2^{7/5}}{2^{5/5}} = 2^{7/5 - 5/5} \\ &= \boxed{2^{2/5}} \end{aligned}$$

$$\begin{aligned} 10. \quad & \left(x^{\frac{1}{3}}y^{\frac{2}{5}}\right)^4 + 5\left(x^{\frac{5}{15}}y^{\frac{6}{15}}\right)^4 \\ &= x^{4/3}y^{8/5} + 5\left(x^{\frac{20}{15^3}}y^{\frac{24}{15^5}}\right) \\ &= x^{4/3}y^{8/5} + 5x^{4/3}y^{8/5} \\ &= x^{4/3}y^{8/5}(1+5) \\ &= \boxed{6x^{4/3}y^{8/5}} \end{aligned}$$

Part III – Simplifying radicals

- If the expression is a fraction, simplify the fraction under the radical as much as possible.
- All factors must be represented using prime numbers.
- The exponents of the prime numbers or variables must be less than or equal to the root or index of the radical.
- If the root and the power of a number/ variable are the same, that variable or number must be brought out of the radical.

Example

- Simplify the fraction under the radical
- Represent all numbers using prime factors
- Rewrite numbers/variables if exponents are greater than the root which in this case is 2 (the square root)
- Remove all factors with an exponent of 2
- Remove the radical from the denominator – under the radical, z must have an exponent of 2 so multiply the numerator and denominator by \sqrt{z} , which will give us our final result

$$\sqrt{\frac{20x^3y^2}{9xz^3}} \rightarrow \sqrt{\frac{20x^2y^2}{9z^3}} \rightarrow \sqrt{\frac{2^2 \cdot 5x^2y^2}{3^2z^3}} \rightarrow \sqrt{\frac{2^2 \cdot 5x^2y^2}{3^2z^2z^1}} \rightarrow \frac{2xy\sqrt{5}}{3z\sqrt{z}} \rightarrow \frac{2xy\sqrt{5}}{3z\sqrt{z}} \cdot \frac{\sqrt{z}}{\sqrt{z}} \rightarrow \frac{2xy\sqrt{5z}}{3z^2}$$

Simplify the following:

$$11. \frac{\sqrt[4]{32}}{\sqrt[4]{2}} = \sqrt[4]{16} = 2$$

$$12. \frac{\sqrt[5]{5}}{\sqrt[5]{27}} = \frac{5^{1/5}}{27^{1/5}} \cdot \frac{27^{4/5}}{27^{4/5}} = \frac{5^{1/5} \cdot 27^{4/5}}{27} = \frac{\sqrt[5]{5} (\sqrt[5]{27})^4}{27}$$

$$\begin{aligned}
 13. \quad & \sqrt[3]{16x^4} \\
 &= \sqrt[3]{(2)^4 x^4} \\
 &= \sqrt[3]{2^3 \cdot 2 \cdot x^3 \cdot x} \\
 &= 2x \sqrt[3]{2x}
 \end{aligned}$$

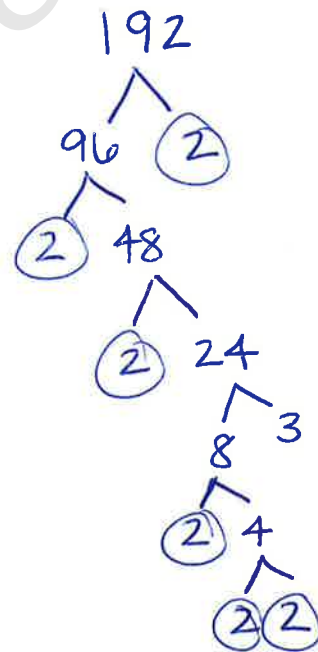
$$\begin{aligned}
 14. \quad & \frac{\sqrt[3]{64x^3y}}{4x^{-3}y} = \frac{\sqrt[3]{(4)^3 x^3 y}}{4x^{-3}y} \\
 &= \frac{\cancel{4} x^3 \sqrt[3]{y}}{\cancel{4} x^{-3} y} = \frac{x^3 \cdot x^3 \sqrt[3]{y}}{y} \\
 &= \frac{x^6 \sqrt[3]{y}}{y}
 \end{aligned}$$

$$15. \quad \frac{\sqrt[4]{192x^3y^7}}{\sqrt[4]{6xy^{-2}}} = \frac{\sqrt[4]{2^6 \cdot 3 \cdot x^3 \cdot y^4 \cdot y^3}}{\sqrt[4]{2 \cdot 3 \cdot x \cdot y^{-2}}}$$

$$= \sqrt[4]{\frac{2^6 \cdot \cancel{3} \cdot x^3 \cdot y^4 \cdot y^3 \cdot y^2}{2 \cdot \cancel{3} \cdot x}}$$

$$= \sqrt[4]{2 \cdot 2^4 \cdot x^2 \cdot y^4 \cdot y^4 \cdot y}$$

$$= 2y^2 \sqrt[4]{2x^2y}$$



Part IV - Like Radicals

- Like radicals have the same radicand and the same index.
- If you have like radicals, then they may be added and/or subtracted just like any other like term. Before you do anything with these expressions, simplify each term individually to its simplest form.
- Then, you treat the radicals like you would any variable ... when you are adding or subtracting, you are "counting" how many like radicals you have.

Example

- Simplify the second term by representing 512 as prime factors.
- Now you have "like radicals". How many do you have?

$$12\sqrt{2} - 7\sqrt{512} \rightarrow 7\sqrt{512} = 7\sqrt{2^9} = 7 \cdot 2 \cdot 2\sqrt{2} = 28\sqrt{2} \rightarrow 12\sqrt{2} - 28\sqrt{2} \rightarrow -16\sqrt{2}$$

Simplify the following:

16. $2\sqrt[4]{1250} - 8\sqrt[4]{32}$

$$2\sqrt[4]{5^4 \cdot 2} - 8\sqrt[4]{2^4 \cdot 2}$$

$$2 \cdot 5\sqrt[4]{2} - 2 \cdot 8\sqrt[4]{2}$$

$$10\sqrt[4]{2} - 16\sqrt[4]{2}$$

$$\boxed{-6\sqrt[4]{2}}$$

Handwritten prime factorization for 1250: $1250 = 625 \cdot 2 = 5^4 \cdot 2$.
Handwritten prime factorization for 32: $32 = 4 \cdot 8 = 2^2 \cdot 2^3 = 2^5$.

17. $5\sqrt[3]{48} - \sqrt[3]{750}$

$$5\sqrt[3]{2^4 \cdot 3} - \sqrt[3]{5^3 \cdot 2 \cdot 3}$$

$$5 \cdot 2\sqrt[3]{2 \cdot 3} - 5\sqrt[3]{2 \cdot 3}$$

$$10\sqrt[3]{6} - 5\sqrt[3]{6}$$

$$\boxed{5\sqrt[3]{6}}$$

Handwritten prime factorization for 48: $48 = 24 \cdot 2 = 3 \cdot 8 \cdot 2 = 3 \cdot 2^3 \cdot 2 = 3 \cdot 2^4$.
Handwritten prime factorization for 750: $750 = 50 \cdot 15 = 5 \cdot 10 \cdot 3 \cdot 5 = 5^3 \cdot 2 \cdot 3$.

18. $\frac{3}{5}\sqrt[3]{5} - \frac{1}{5}\sqrt[3]{625}$

$$\frac{3}{5}\sqrt[3]{5} - \frac{1}{5}\sqrt[3]{5^4}$$

$$\frac{3}{5}\sqrt[3]{5} - \sqrt[3]{5}$$

$$\boxed{-\frac{2}{5}\sqrt[3]{5}}$$

Handwritten prime factorization for 625: $625 = 25 \cdot 25 = 5^2 \cdot 5^2 = 5^4$.