

LEARNING TARGET: I will perform operations on complex numbers

Imaginary Numbers

Does $x^2 = -1$ have any solutions?

$\sqrt{-1}$ is not a real number – so there are no real number solutions because we are unable to take the square root of a negative and get a real number. However, quadratics have the same number of solutions as the leading power – so this equation would have 2 solutions.

In order to overcome this, mathematicians created an expanded system of numbers using the imaginary unit i .

See if we can develop a shortcut:
 ① Divide by 2
 - even +
 - odd -

Defined as: $i = \sqrt{-1}$, note therefore, $i^2 = -1$

On board:
 $i, i^2, i^3, i^4, i^5, i^6$
 $i, -1, -i, 1, i, -1$
 cycle
 $i^{54} = -1$

Using this definition, we are able to simplify powers of i

② if R then i

$i^5 = i$ $i^{26} = -1$ $i^{122} = -1$ $i^{54} = -1$

$5 \div 2 = 2R1$ $26 \div 2 = 13$ $122 \div 2 = 61$ $54 \div 2 = 27$

The imaginary unit i can be used to write the square root of any negative number

If r is a positive real number, then $\sqrt{-r} = i\sqrt{r}$

For example:

$\sqrt{-3} = i\sqrt{3}$ $\sqrt{-28} = 2i\sqrt{7}$
 $i\sqrt{28} = i\sqrt{4}\sqrt{7} = 2i\sqrt{7}$

Solve the following equations:

$x^2 = -13$
 $x = \pm\sqrt{-13}$
 $x = \pm i\sqrt{13}$

$3x^2 - 7 = -31$
 $+7 \quad +7$

 $3x^2 = -24$
 $\frac{3x^2}{3} = \frac{-24}{3}$
 $x^2 = -8$
 $x = \pm\sqrt{-8}$
 $x = \pm 2i\sqrt{2}$

$x^2 - 8 = -36$
 $+8 \quad +8$

 $x^2 = -28$
 $x = \pm\sqrt{-28}$
 $x = \pm 2i\sqrt{7}$

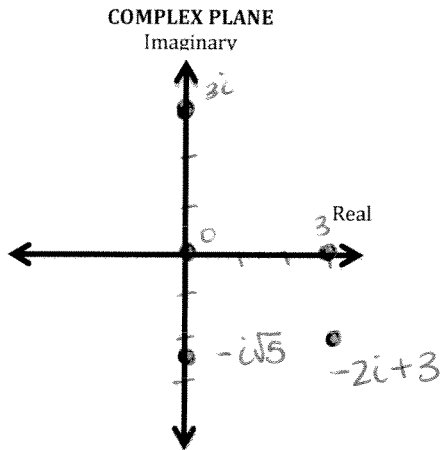
Complex Numbers

A complex number is a number consisting of a real part and an imaginary part. A complex number is an element of the complex plane.

"Real Part" Imaginary Part

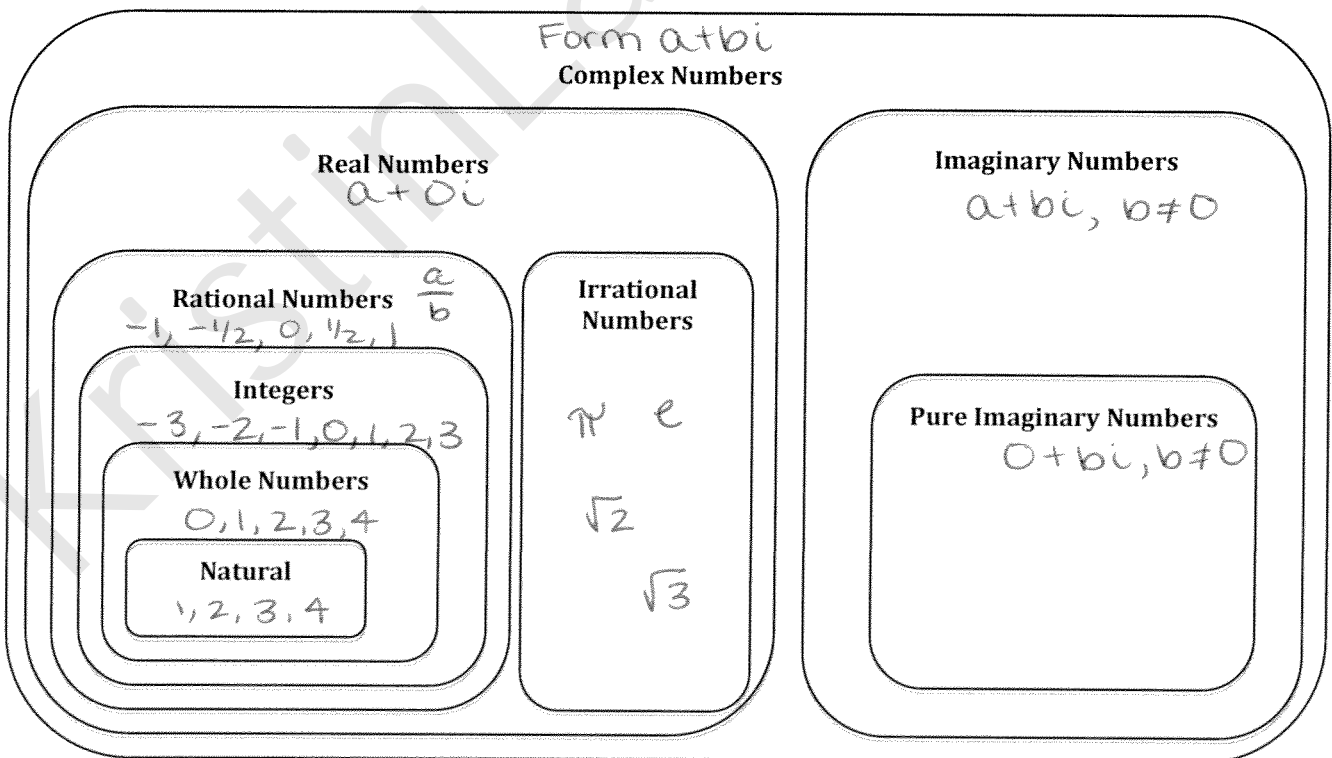
Written in standard form: $a + bi$ where a and b are real numbers

Write the following in standard form and plot on the complex plane:



- a) 3
- b) $3i$
- b) 0
- d) $-i\sqrt{5}$
- e) $-2i + 3$

Complex numbers are organized into a hierarchy of subsets:



Operations with Complex Numbers

To add or subtract complex numbers, add and subtract their real and imaginary parts separately.

By Definition:

$$\text{Sum: } (a + bi) + (c + di) = (a + c) + (b + d)i$$

$$\text{Difference: } (a + bi) - (c + di) = (a - c) + (b - d)i$$

$$(8 - i) + (5 + 4i)$$

$$(8+5) + (-i+4i)$$

$$\boxed{13 + 3i}$$

$$(7 - 6i) - (3 - 6i)$$

$$(7-3) + (-6i - -6i)$$

$$4 + 0$$

$$\boxed{4}$$

$$10 - (6 + 7i) + 4i$$

$$[(10-6) - 7i] + 4i$$

$$(4 - 7i) + 4i$$

$$\boxed{4 - 3i}$$

To multiply complex numbers, use the distribution property, or FOIL method

$$4i(-6 + i)$$

$$-24i + 4i^2$$

$$-24i + 4(-1)$$

$$-24i - 4$$

$$\boxed{-4 - 24i}$$

$$(9 - 2i)(-4 + 7i)$$

$$-36 + 63i + 8i - 14i^2$$

$$-36 + 71i - 14(-1)$$

$$\boxed{-22 + 71i}$$

$$(3 - 5i)(3 + 5i)$$

$$9 + 15i - 15i - 25i^2$$

$$9 - 25(-1)$$

$$\boxed{34}$$

$$(9 - 6i)(9 + 6i)$$

$$81 + 54i - 54i - 36i^2$$

$$81 - 36(-1)$$

$$\boxed{117}$$

Do you notice anything about the above two examples?

The product of complex conjugates is always a real number.

Numbers of the form $a + bi$ and $a - bi$ are called complex conjugates and the product of complex conjugates is always a real number.

This fact is used to write the quotient of two complex numbers in standard form.

Must write in standard form

$$\frac{-13 + 33i}{17}$$

$$-\frac{13}{17} + \frac{33}{17}i$$

$$\frac{7 + 5i}{1 - 4i} \cdot \frac{(1 + 4i)}{(1 + 4i)} = \frac{7 + 28i + 5i + 20i^2}{1 + 4i - 4i - 16i^2}$$

$$\frac{7 + 33i + 20(-1)}{1 - 16(-1)}$$

Field Properties of Complex Numbers

Commutative Property for Addition and Multiplication

$$(a + bi) + (c + di) = (c + di) + (a + bi)$$

SAME as with real #s

$$(3 + 2i) + (1 + 4i) = (1 + 4i) + (3 + 2i)$$

$$4 + 6i = 4 + 6i$$

$$(a + bi)(c + di) = (c + di)(a + bi)$$

Additive and Multiplicative Identity

$$(a + bi) + (0 + 0i) = (a + bi)$$

$$(a + 0) + (bi + 0i) = a + bi$$

$$a + bi = a + bi$$

$$(a + bi)(1 + 0i) = (a + bi)$$

$$a(1) + a(0i) + bi(1) + bi(0i)$$

$$a + 0 + bi + 0 = a + bi$$

$$a + bi = a + bi$$

Additive and Multiplicative Inverses

$$(a + bi) + (-a - bi) = 0$$

$$(a - a) + (bi - bi)$$

$$0 + 0 = 0$$

$$(a + bi) \left(\frac{1}{a + bi} \right) = 1$$

Non-Zero Product Property

This property states that if both complex numbers are non-zero, the product of the two complex numbers cannot be zero.