LEARNING TARGET: I will perform operations on complex numbers

Imaginary Numbers

Does $x^2 = -1$ have any solutions?

$\sqrt{-1}$ is not a real number – so there are no real number solutions because we are unable to take the square root of a negative and get a real number. However, quadratics have the same number of solutions as the leading power – so this equation would have 2 solutions.

See if we can develop shortcut:
① Divide by 2
- even +
- odd −
② if R then i

In order to overcome this, mathematicians created an expanded system of numbers using the imaginary unit i.

On board:
$i, i^2, i^3, i^4, i^5, i^6$
$i, -1, -i, 1, i, -1$
cycle

Defined as: $i = \sqrt{-1}$, note therefore, $i^2 = -1$

Using this definition, we are able to simplify powers of i

$i^5 = i$ $\qquad$ $i^{26} = -1$ $\qquad$ $i^{122} = -1$ $\qquad$ $i^{54} = -1$

$5 \div 2 = 2R1$ $\qquad$ $26 \div 2 = 13$ $\qquad$ $122 \div 2 = 61$ $\qquad$ $54 \div 2 = 27$

The imaginary unit i can be used to write the square root of <u>*any*</u> negative number

If r is a positive real number, then $\sqrt{-r} = i\sqrt{r}$

For example:

$\sqrt{-3} = i\sqrt{3}$ $\qquad$ $\sqrt{-28} = 2i\sqrt{7}$

$i\sqrt{28} = i\sqrt{4}\sqrt{7} = 2i\sqrt{7}$

Solve the following equations:

$x^2 = -13$
$x = \pm\sqrt{-13}$
$x = \pm i\sqrt{13}$

$3x^2 - 7 = -31$
$+7 \quad +7$
$\frac{3x^2}{3} = \frac{-24}{3}$
$x^2 = -8$
$x = \pm\sqrt{-8}$
$x = \pm 2i\sqrt{2}$

$x^2 - 8 = -36$
$+8 \quad +8$
$x^2 = -28$
$x = \pm\sqrt{-28}$
$x = \pm 2i\sqrt{7}$

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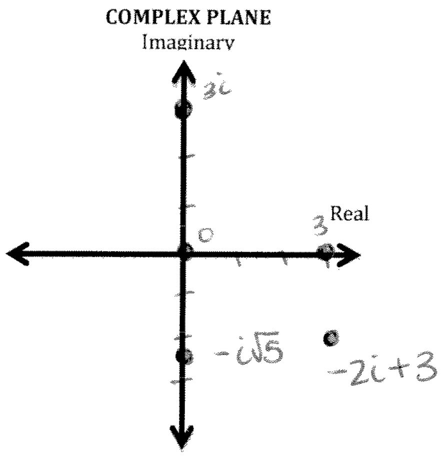
Complex Numbers

A complex number is a number consisting of a real part and an imaginary part. A complex number is an element of the complex plane.

"Real Part" Imaginary Part

Written in standard form: $a + bi$ where a and b are real numbers

Write the following in standard form and plot on the complex plane:



a) 3

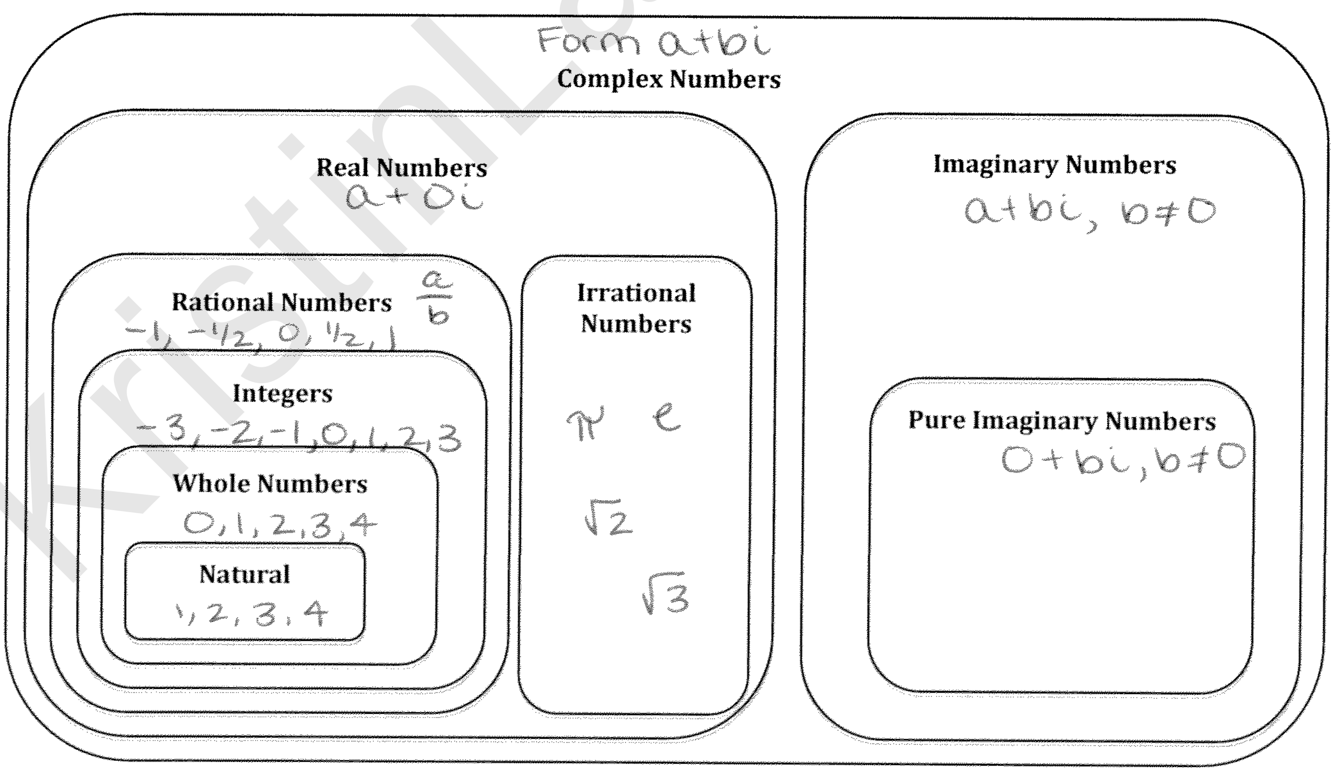
b) $3i$

b) 0

d) $-i\sqrt{5}$

e) $-2i + 3$

Complex numbers are organized into a hierarchy of subsets:



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Operations with Complex Numbers

To add or subtract complex numbers, add and subtract their real and imaginary parts separately.

By Definition:

$$Sum: (a + bi) + (c + di) = (a + c) + (b + d)i$$

$$Difference: (a + bi) - (c + di) = (a - c) + (b - d)i$$

$(8 - i) + (5 + 4i)$

$(8+5) + (-i+4i)$

$\boxed{13 + 3i}$

$(7 - 6i) - (3 - 6i)$

$(7-3) + (-6i - -6i)$

$4 + 0$

$\boxed{4}$

$10 - (6 + 7i) + 4i$

$[(10-6) - 7i] + 4i$

$(4 - 7i) + 4i$

$\boxed{4 - 3i}$

To multiply complex numbers, use the distribution property, or FOIL method

$4i(-6 + i)$

$-24i + 4i^2$

$-24i + 4(-1)$

$-24i - 4$

$\boxed{-4 - 24i}$

$(9 - 2i)(-4 + 7i)$

$-36 + 63i + 8i - 14i^2$

$-36 + 71i - 14(-1)$

$\boxed{-22 + 71i}$

$(3 - 5i)(3 + 5i)$

$9 + 15i - 15i - 25i^2$

$9 - 25(-1)$

$\boxed{34}$

$(9 - 6i)(9 + 6i)$

$81 + 54i - 54i - 36i^2$

$81 - 36(-1)$

$\boxed{117}$

Do you notice anything about the above two examples?

The product of complex conjugates is always a real number.

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Numbers of the form $a + bi$ and $a - bi$ are called complex conjugates and the product of complex conjugates is ***always*** a real number.

This fact is used to write the quotient of two complex numbers in standard form.

$$\frac{7+5i}{1-4i}\cdot\frac{(1+4i)}{(1+4i)} = \frac{7+28i+5i+20i^2}{1+4i-4i-16i^2}$$

$$\frac{7+33i+20(-1)}{1-16(-1)}$$

$$\frac{-13+33i}{17}$$

Must write in Standard form

$$\frac{-13}{17} + \frac{33}{17}i$$

Field Properties of Complex Numbers

Commutative Property for Addition and Multiplication

$(a + bi) + (c + di) = (c + di) + (a + bi)$

$(a + bi)(c + di) = (c + di)(a + bi)$

SAME as with Real #'s

$(3+2i)+(1+4i)=(1+4i)+(3+2i)$

$4+6i = 4+6i$ ✓

Additive and Multiplicative Identity

$(a + bi) + (0 + 0i) = (a + bi)$

$(a+0)+(bi+0i)= a+bi$

$a+bi = a+bi$ ✓

$(a + bi)(1 + 0i) = (a + bi)$

$a(1)+a(0i)+bi(1)+bi(0i)$

$a+0+bi+0 = a+bi$

$a+bi = a+bi$ ✓

Additive and Multiplicative Inverses

$(a + bi) + (-a - bi) = 0$

$(a-a)+(bi-bi)$

$0 + 0 = 0$ ✓

$(a + bi)\left(\frac{1}{a + bi}\right) = 1$ ✓

Non-Zero Product Property

This property states that if both complex numbers are non-zero, the product of the two complex numbers cannot be zero.

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