LEARNING TARGET: I will solve absolute value equations and inequalities algebraically and graphically

Set Builder Notation

Set builder notation is a mathematical notation for describing a set by stating the properties that must be satisfied in order to be a member of the set.

Let a and b be real numbers such that $a < b$.		
Interval Notation	**Set-Builder Notation**	**Graph**
(a, b)	$\{x \mid a < x < b\}$	a b x
$[a, b]$	$\{x \mid a \le x \le b\}$	a b x
$[a, b)$	$\{x \mid a \le x < b\}$	a b x
$(a, b]$	$\{x \mid a < x \le b\}$	a b x
(a, ∞)	$\{x \mid x > a\}$	a x
$[a, \infty)$	$\{x \mid x \ge a\}$	a x
$(-\infty, b)$	$\{x \mid x < b\}$	b x
$(-\infty, b]$	$\{x \mid x \le b\}$	b x
$(-\infty, \infty)$	$\{x \mid x \text{ is a real number}\}$ or $\mathbb{R}$ (set of all real numbers)	x

What are all of the elements of the following sets?

$\{x|0 < x < 6\}$ $\{x| -2 \le x \le 3\}$ $\{y|4 < y \le 10\}$

$\{1, 2, 3, 4, 5\}$ $\{-2, -1, 0, 1, 2, 3\}$ $\{5, 6, 7, 8, 9, 10\}$

How many ways can you represent the following set using set builder notation?

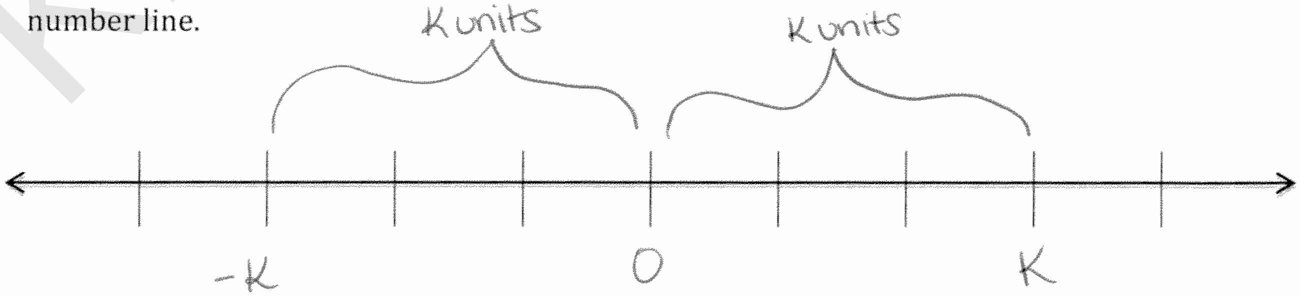
$\{2,3,4,5,6\}$

$\{x \mid 2 \le x \le 6\}$ $\{x \mid 2 \le x < 7\}$ all represent the

$\{x \mid 1 < x \le 6\}$ $\{x \mid 1 < x < 7\}$ Same set.

Absolute Value

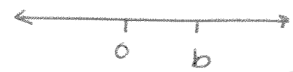
The absolute value of a number x, written $|x|$, is the distance the number is from 0 on a number line.



Which can be written as:

$$|x| = \begin{cases} x, \text{ if } x \text{ is positive} \\ 0, \text{ if } x = 0 \\ -x, \text{ if } x \text{ is negative} \end{cases}$$

$b = |a| \therefore a = b$
$a = -b$



Additionally:

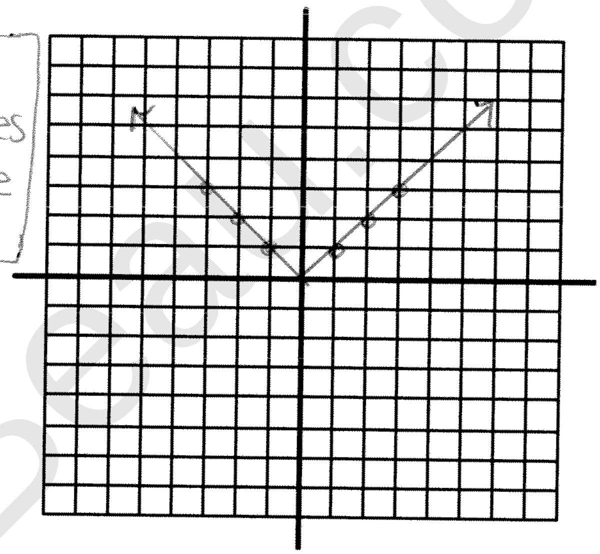
$\boxed{\text{For any real numbers } a \text{ and } b, \text{ where } b \geq 0, \text{ if } |a| = b, \text{ then } a = b \text{ or } a = -b}$

Discuss Meaning

What does this look like graphically?

x	$y = \lvert x \rvert$
-3	3
-2	2
-1	1
0	0
1	1
2	2
3	3

Knowing about absolute value - does this graph make sense?



Characteristics of the absolute value graph:

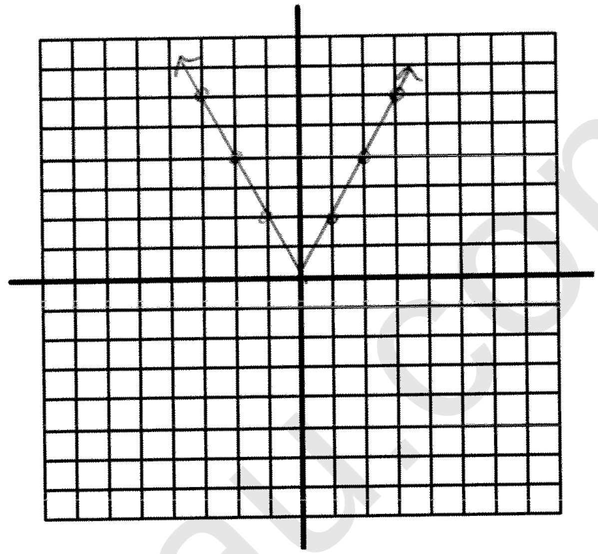
- 'V' shape
- Symmetrical about vertical axis (or line)

Absolute Value Transformations [Small Group Work]

On each graph next to the form of the absolute value function, sketch both the parent function and the stated form.

What about the form $y = a|x|$?

x	$y = 2|x|$
-3	6
-2	4
-1	2
0	0
1	2
2	4
3	6



1. Describe the difference between the graph of the parent function and the graph of $y = 2|x|$

   – narrower

2. What is the slope of the right side of the parent function? The left side?

   R: +1 L: -1

3. What is the slope of the right side of $y = 2|x|$? The left side?

   R: +2 L: -2

4. Based on what you observed from the two functions above, what do you think the value of *a* controls in the absolute value function?

   a controls the slope of each side.

5. Graph $y = |x|$ and $y = -|x|$ on the graphing calculator. What do you notice?

   If $a > 0$, then faces upward

   If $a < 0$, then faces downward

Use a graphing calculator to complete the table.

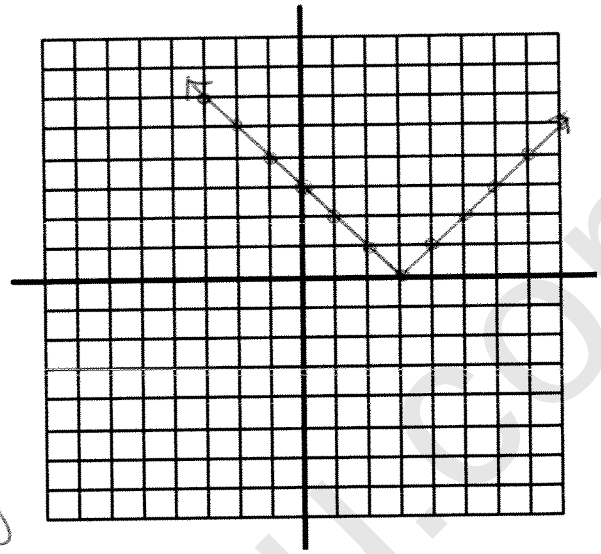
Equation	What I ***think*** will happen	What ***actually*** happened	Sketch of the graph	How is it different from parent function?
$y = 3\lvert x\rvert$		gets narrower		
$y = \frac{1}{3}\lvert x\rvert$		gets wider		
$y = -\frac{1}{2}\lvert x\rvert$		gets wider faces down		
$y = -2\lvert x\rvert$		gets narrower + faces down		

Summarize the effect that a has on the graph of $y = a\lvert x\rvert$.

a is the slope of the branches of the parent function and determines if the graph faces upward or downward.

What about the form $y = |x - h|$?

x	$y = \|x - 3\|$
-3	6
-2	5
-1	4
0	3
1	2
2	1
3	0



1. Describe the difference between the parent function (the equation – not the graph) and $y = |x - 3|$.

   there is shift occuring

2. What happened to the graph of the parent function as a result of this difference?

   the parent graph shifted over 3 units

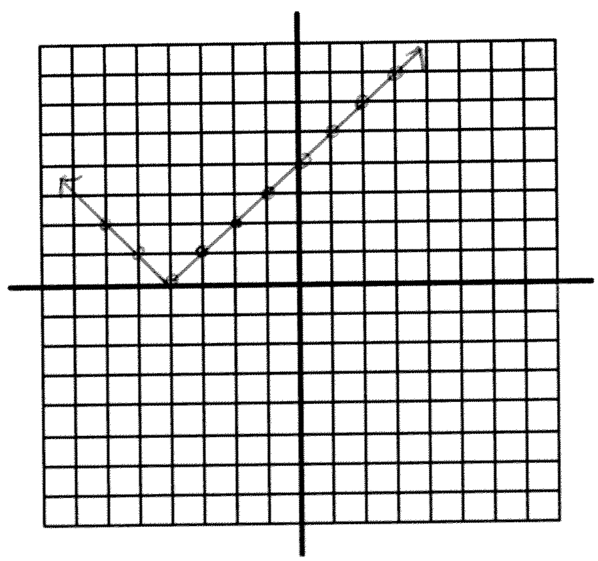
3. What number replaces h in $y = |x - 3|$?

   3

4. Name the coordinates of the vertex of this graph.

   (3,0)

x	$y = \|x + 4\|$
-3	1
-2	2
-1	3
0	4
1	5
2	6
3	7



5. What number replaces h in $y = |x + 4|$?

   −4

6. Name the coordinates of the vertex of the graph of $y = |x + 4|$.

   (−4,0)

Use a graphing calculator to complete the table below.

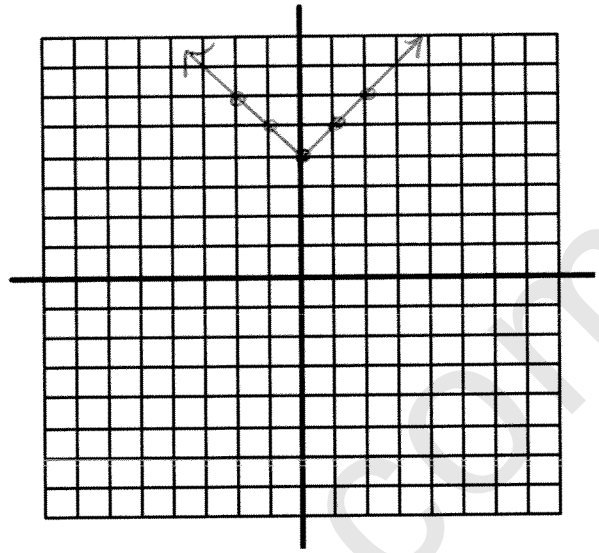
Equation	What I ***think*** will happen	What ***actually*** happened	Sketch of the graph	How is it different from parent function?
$y = \|x+2\|$		shift L 2		
$y = \|x-5\|$		shift R 2		
$y = \|x+1\|$		shift L 1		
$y = \|2x-3\|$		shift R 3 + steeper		

Summarize the effect that "h" has on the graph of $y = |x-h|$. Be sure to include what happens when $h<0$ and $h>0$.

h is a horizontal shift. if h > 0 it is a shift to the right if h < 0 it is a shift to the left

What about the form $y = |x| + k$?

x	$y = \|x\| + 4$
-3	7
-2	6
-1	5
0	4
1	5
2	6
3	7



1. Describe the difference between $y = |x| + 4$ and the parent function.

   the + 4 after the absolute value

2. What happened to the graph of the parent function as a result of this difference?

   the graph shifted up 4 units

3. What number replaces "k" in $y = |x| + 4$?

   4

4. Name the coordinates of the vertex of the graph of $y = |x| + 4$.

   (0, 4)

Use a graphing calculator to complete the table.

Equation	What I ***think*** will happen	What ***actually*** happened	Sketch of the graph	How is it different from parent function?
$y = \|x\| + 3$		Shift up 3 units		
$y = \|x\| - 2$		Shift down 3 units		
$y = \|x\| + 1$		Shift up 1 unit		
$y = 2\|x - 3\| - 4$		Shift down 4 R, 3 + become narrower		

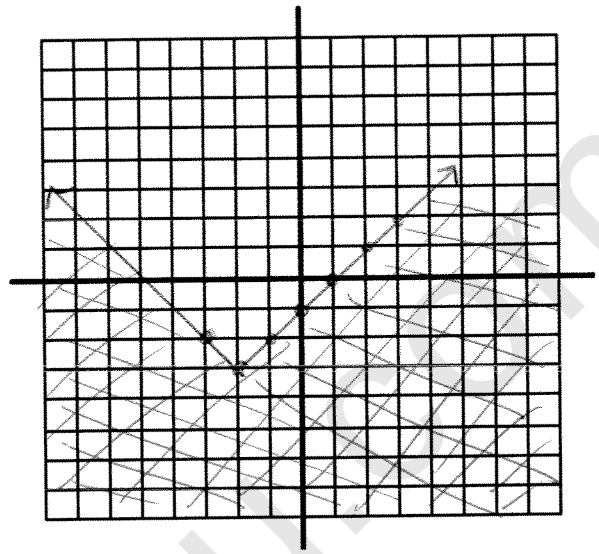
Summarize the effect that "k" has on the graph of $y = |x| + k$. Make sure you include what happens when $k < 0$ and $k > 0$.

K is a vertical shift. When K<0 the graph shifts down, when K>0 the graph shifts up.

Graphing Absolute Value Inequalities

$y \leq |x+2| - 3$

x	$y \leq \lvert x+2 \rvert - 3$
-3	−2
-2	−3
-1	−2
0	−1
1	0
2	1
3	2



STEPS:

1. Graph as if it were an equation ….
   (< or >) would be graphed with a dotted line
2. Shade above the graph if $>$ or $\geq$
3. Shade below the graph if $<$ or $\leq$

Writing Equations of Absolute Value Functions

In Summary: $y = a|x-h| + k$

- (h, k) is the vertex of the graph
- a controls the direction of opening and the slope of each side of the graph.
    - $a > 0$ Graph opens up
    - $a < 0$ Graph opens down

Write the equation of the absolute value function with the given vertex, slope and direction of opening

Vertex $(-3, 1)$; slope ± 2; direction of opening: down

$y = -2|x+3| + 1$

Vertex $(6, -7)$; slope $\pm 1/2$; direction of opening: up

$y = \frac{1}{2}|x-6| - 7$

Vertex $(2, 5)$; slope ± 1; direction of opening: up

$y = |x-2| + 5$

Vertex $(-2, -9)$; slope $\pm 1/3$; direction of opening; down

$y = -\frac{1}{3}|x+2| - 9$

Absolute Value Equations

STEPS TO SOLVE ALGEBRAICALLY:

1. Write two equations (+/-)
2. Solve each equation
3. Check each equation in the *original* absolute value equation

When solving absolute value equations, it is possible to get extraneous solutions.
Extraneous solutions do not satisfy the original equation, and therefore must be rejected.

Show how to solve problem graphically on calc

Solve the following absolute value equations

$|x-5|=7$

$x-5=7$, $+5 \quad +5$, $x=12$

$x-5=-7$, $+5 \quad +5$, $x=-2$

$|12-5|=7$, $7=7$ ✓

$|-2-5|=7$, $7=7$ ✓

$|5x-10|=45$

$5x-10=45$, $+10 \quad +10$, $\frac{5x}{5}=\frac{55}{5}$, $x=11$

$|5(11)-10|=45$, $45=45$ ✓

$5x-10=-45$, $+10 \quad +10$, $\frac{5x}{5}=\frac{-35}{5}$, $x=-7$

$|5(-7)-10|=45$, $|-45|=45$ ✓

$|2x+12|=4x$

$2x+12=4x$, $-2x \quad -2x$, $\frac{12}{2}=\frac{2x}{2}$, $6=x$

$|2(6)+12|=4(6)$, $24=24$ ✓

$2x+12=-4x$, $-2x \quad -2x$, $\frac{12}{-6}=\frac{-6x}{-6}$, $-2=x$

$|2(-2)+12|=4(-2)$, $8 \neq -8$

Extraneous!

TRY ON OWN

$|4x-1|=2x+9$

* careful when writing (-) branch

$4x-1=2x-9$, $\frac{2x}{2}=\frac{10}{2}$, $x=5$ ✓

$4x-1=-2x-9$, $\frac{6x}{6}=\frac{-8}{6}$, $x=\frac{-4}{3}$

$|4(-\frac{4}{3})-1|=2(\frac{4}{3})+9$

$6\frac{1}{3}=6\frac{1}{3}$ ✓

Absolute Value Inequalities

Absolute value inequalities can be solved by rewriting the problems as compound inequalities and then solving each part.

Absolute Value Inequalities

Inequality	Equivalent form	Graph of solution
$\lvert ax+b\rvert < c$	$-c < ax+b < c$	
$\lvert ax+b\rvert \le c$	$-c \le ax+b \le c$	
$\lvert ax+b\rvert > c$	$ax+b < -c$ or $ax+b > c$	
$\lvert ax+b\rvert \ge c$	$ax+b \le -c$ or $ax+b \ge c$	

These are the forms

DOES THIS MAKE SENSE?

RECALL: Open circle vs. closed circle

When multiplying or dividing by a negative – reverse the direction of the inequality

SHOW ON CALC

Solve the following absolute value inequalities, then graph the solution.

① Determine form
② Solve
③ Check

$|4x+5| > 13$

$4x+5 < -13$, $-5\ -5$, $\frac{4x}{4} < \frac{-18}{4}$, $x < -\frac{9}{2}$

$4x+5 > 13$, $-5\ -5$, $\frac{4x}{4} > \frac{8}{4}$, $x > 2$

$|3x+5| \ge 10$

$3x+5 \le -10$, $-5\ -5$, $\frac{3x}{3} \le \frac{-15}{3}$, $x \le -5$

$3x+5 \ge 10$, $-5\ -5$, $\frac{3x}{3} \ge \frac{5}{3}$, $x \ge \frac{5}{3}$

What does tolerance mean?
- maximum acceptable deviation of an item from some measurement or ideal

Real World Applications

A professional baseball should weigh 5.125 ounces, with a *tolerance* of 0.125 ounce. Write and solve an absolute value inequality that describes the acceptable weights for a baseball. Express your answer using set builder notation.

① VERBAL MODEL

$|\text{weight} - \text{ideal weight}| \leq \text{tolerance}$

$|w - 5.125| \leq 0.125$

$-0.125 \leq w - 5.125 \leq 0.125$
$+5.125 \quad\quad +5.125 \quad\quad +5.125$

$5 \leq w \leq 5.25$

$\{w \mid 5 \leq w \leq 5.25\}$

The thickness of the mats used in the rings, parallel bars, and vault events must be between 7.5 inches and 8.25 inches, inclusive. Write an absolute value inequality describing the acceptable mat thicknesses.

① What is the question asking?
- write an absolute value inequality

② Do we have all our info needed?
- we need 'ideal' like in last problem
  - we need to find the mean.

$\frac{7.5 + 8.25}{2} = 7.875 \quad \text{tolerance} = 8.25 - 7.875 = 0.375$

③ write verbal model

$|\text{actual thickness} - \text{ideal/mean}| \leq \text{tolerance}$

$|t - 7.875| \leq 0.375$